3

Geometry and Measurement

There are many challenges to deal with when designing an airport. Number one is to build the runways so that airplanes do not crash into each other! In this chapter, you will see that airport designers need to know about lines and angles, and about calculating area. All of this math adds up to keeping air travellers safe and happy.

What You Will Learn

- to draw a line segment parallel to another line segment
- to draw a line segment perpendicular to another line segment
- to draw a line that divides a line segment in half and is perpendicular to it
- to divide an angle in half
- to develop and use formulas to calculate the areas of triangles and parallelograms

Key Words

parallel
perpendicular
perpendicular bisector
angle bisector
parallelogram
base
height

MATH LINK

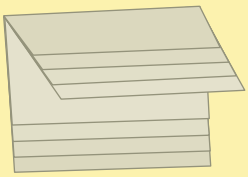
Throughout the chapter, you will work on a design for your own airport runway system. To do this, you will draw parallel and perpendicular lines and make areas in the shapes of triangles and parallelograms.

FOLDABLES™
Study Tool

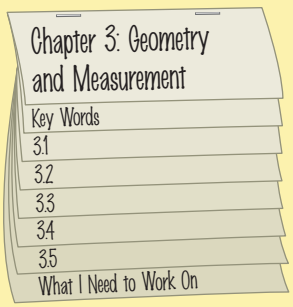
Make the following Foldable to organize what you learn in Chapter 3.

Step 1 Collect four sheets of paper and place them 2 cm apart. Keep the edges straight.

Step 2 Fold the top edge of the paper. Stop 2 cm from the bottom edge. This makes all tabs the same size. Staple together along the fold.



Step 3 Label the tabs.



Literacy Link

As you work through Chapter 3, take notes under the appropriate tab. Include information about the key words, examples, and key ideas.

3.1

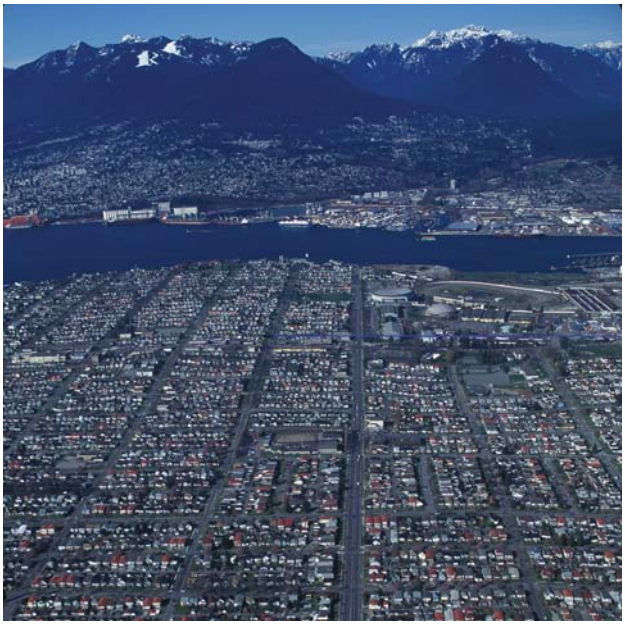
Parallel and Perpendicular Line Segments

Focus on...

After this lesson, you will be able to...

- ❑ draw line segments that are parallel to each other
- ❑ draw line segments that are at right angles to each other

Many city streets have been built using a grid. Even a city like Vancouver, which has an irregular shape, is set up this way. What types of lines do the streets that run side by side form? What types of lines do the streets that cross each other at right angles form?



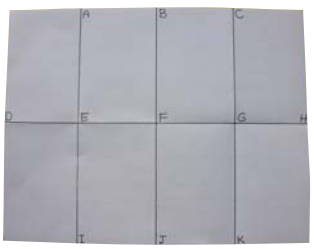
Explore the Math

Materials

- ruler
- protractor

What are parallel and perpendicular line segments?

1. Fold a sheet of blank paper in half, then into quarters, and then into eighths. Unfold the paper and use a ruler and a pencil to draw line segments along the creases. Label the endpoints from A to K, as shown.



2. Identify each pair of line segments that you think are parallel. Use your ruler to measure the perpendicular distance between the line segments. Make two measurements, one at each end. Record the measurements in a table.

3. Identify each pair of line segments that you think are perpendicular. Use your protractor to measure the angles made by these line segments. Record the measurements in a table.

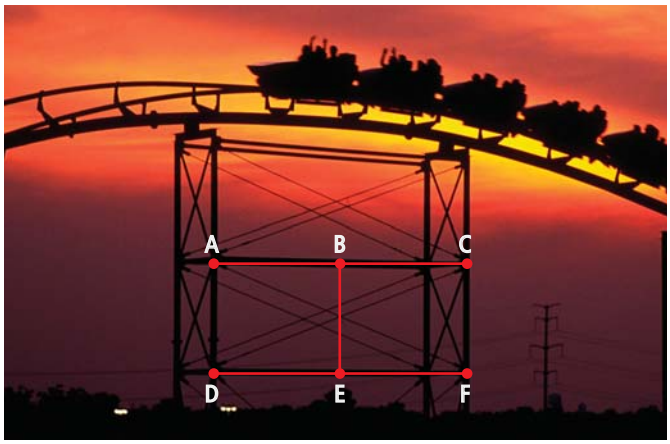
Reflect on Your Findings

4. a) Describe the two measurements you made for each pair of line segments you thought were parallel. What do you think is true about the distance between parallel line segments?

b) What kind of angle did you measure for each pair of line segments you thought were perpendicular? What do you think is true about the angle made by perpendicular line segments?

Example 1: Identify Parallel and Perpendicular Line Segments

Look at the picture of the roller coaster.



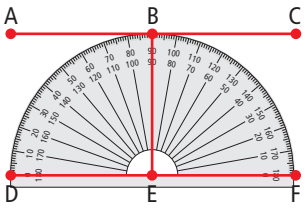
a) Are line segments AC and DF **parallel**?

b) Are line segments DF and BE **perpendicular**?

Solution

a) Measure the perpendicular distance between AC and DF in two locations. If the measurements are the same, AC is parallel to DF.

b) Place a protractor at point E on line segment DF. If ∠BEF or ∠BED measures 90°, DF and BE are perpendicular.



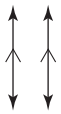
Show You Know

a) Are line segments AC and BE perpendicular? How do you know?

b) Would a line segment joining A to D be parallel to line segment BE? How do you know?

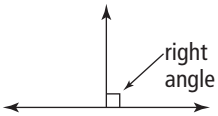
parallel

- describes lines in the same plane that never cross, or intersect
- they are marked using "arrows"



perpendicular

- describes lines that intersect at right angles (90°)
- they are marked using a small square

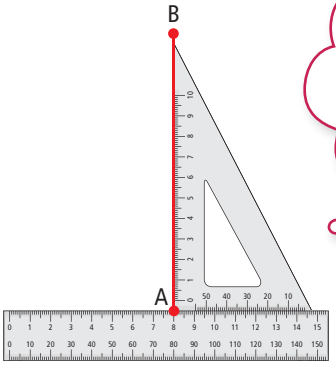


Example 2: Draw Parallel Line Segments

Draw a line segment, AB. Draw another line segment, CD, parallel to AB.

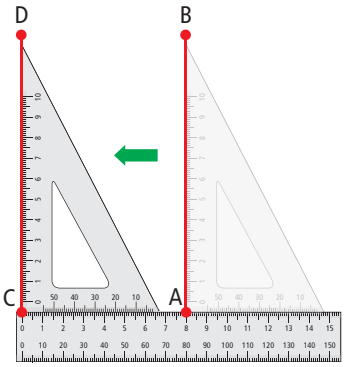
Solution

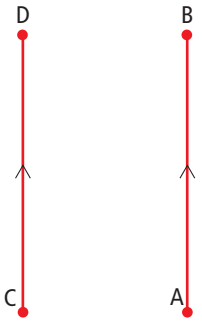
Use a ruler to draw a line segment. Label its endpoints A and B. Place the edge of a right triangle along AB as shown. Place a ruler against the bottom edge of the triangle.



For the right triangle, use a plastic triangle from a math set or cut a right triangle from the corner of a piece of paper.

Slide the triangle along the ruler. Draw along the perpendicular edge of the triangle to create a line parallel to AB. Label the endpoints of the parallel line segment C and D.





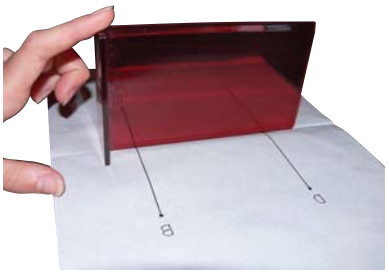
Remember to mark the lines with arrows to show that they are parallel.

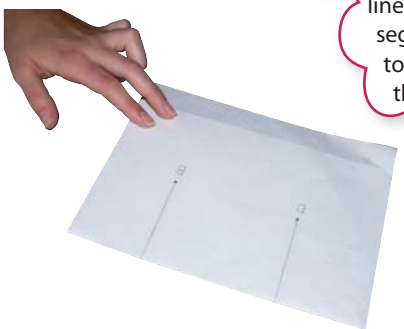
Check:

Verify that line segments are parallel using one of these methods:

- Use a Mira.
- Use paper folding.

Place the Mira across the line segments. If both segments can reflect onto themselves, they are parallel.





Fold across the two line segments. If both segments can lie on top of themselves, they are parallel.

Literacy Link

Right Triangle

It is called a right triangle because it has an angle of 90°.

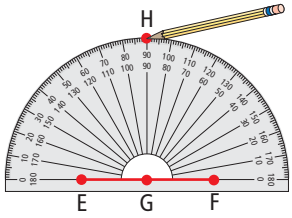
Example 3: Draw Perpendicular Line Segments

Draw a line segment, EF. Draw another line segment, GH, perpendicular to EF.

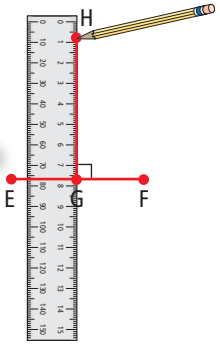
Solution

Use a ruler to draw a line segment.
Label its endpoints E and F.
Mark a point along EF. Label this point G.

Place a protractor at point G. Mark a point that is at right angles to line segment EF.
Label this endpoint H.



Connect points G and H to draw a line segment perpendicular to EF.



Remember to mark the angle with a small square to show that the lines are perpendicular.

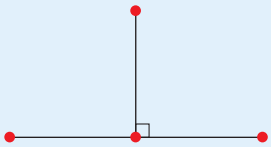
Check:

Verify that line segments are perpendicular using one of these methods:

- Use paper folding.
- Use a Mira.

Key Ideas

- Parallel line segments are line segments in the same plane that do not intersect.
- The perpendicular distance between parallel line segments must be the same at each end of the line segments.
- Some ways to create parallel line segments include
  – using a ruler and a right triangle
  – using paper folding
- Perpendicular line segments are line segments that intersect at 90°.
- Some ways to create perpendicular line segments include
  – using a ruler and a protractor
  – using paper folding



Communicate the Ideas

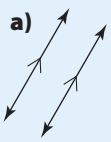
1. a) What are five examples of parallel line segments in the real world? Sketch each example.

b) What are five examples of perpendicular line segments in the real world? Sketch each example.

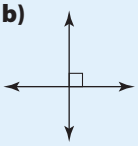
c) Share your lists and sketches with a partner.

2. Are each of the following pairs of lines and line segments parallel, perpendicular, or neither? Explain how you know.

a)



b)



c)

d)

3. a) What are two methods you might use to draw parallel line segments?

b) What do you know about parallel line segments that helps you to draw them?

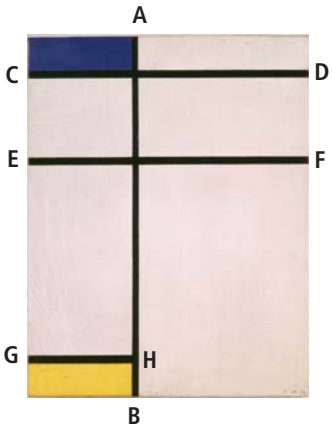
4. a) What are two methods you might use to draw perpendicular line segments?

b) What do you know about perpendicular line segments that helps you to draw them?

Practise

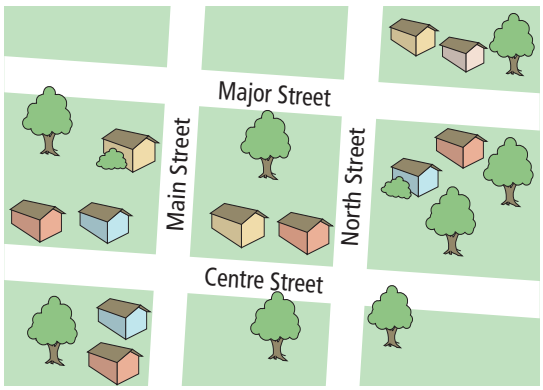
For help with #5 and #6, refer to Example 1 on page 83.

5. What are the parallel and perpendicular line segments in the painting?



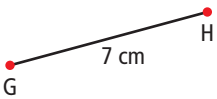
Composition in Black, Blue, Yellow, and White. 1936. Mondrian, Piet (1872–1944)

6. Identify the parallel and perpendicular streets in the diagram.



For help with #7 and #8, refer to Examples 2 and 3 on pages 84–85.

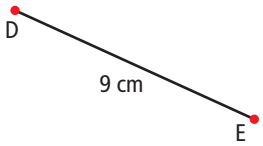
7. On a piece of paper, draw a 7-cm line segment as shown.



a) Draw two line segments that are parallel to GH.

b) On a separate piece of paper, draw GH again. Draw two line segments that are perpendicular to GH.

8. Draw a 9-cm line segment as shown.



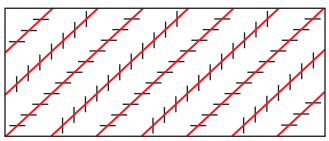
a) Draw two line segments that are parallel to DE.

b) Draw DE again. Draw two line segments that are perpendicular to DE.

Apply

9. Are the wings of this biplane parallel or perpendicular? How do you know?

10. Are the red line segments on this rectangle parallel? Explain how to verify your answer.



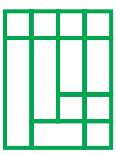
11. Name and sketch at least two objects in the real world that include

a) parallel lines

b) perpendicular lines

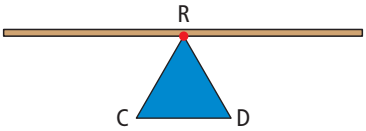
c) both parallel and perpendicular lines

12. The drawing shows part of a Chinese lattice design. Copy the design, using only parallel and perpendicular line segments.



13. Dakota has a shelf on her bedroom wall for her soccer trophies. She wants to hang another shelf parallel to it. Use what you know about drawing parallel line segments to explain how Dakota can do this.

14. You are designing a seesaw. You need to place a perpendicular support from R to CD. Copy the diagram and draw the support.

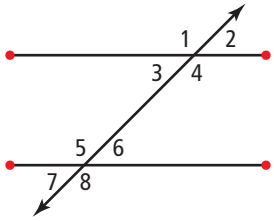


Extend

15. Using parallel and perpendicular lines only, make a copy of this letter *F* that is 5 cm high. Draw the reflection of your letter.

Refer to Chapter 1 for more about reflections.

16. Draw parallel line segments and a third line that intersects them.



a) Measure angles 1 to 8 and write each measurement inside the angle.

b) What do you notice about the angles?

MATH LINK

You are going to create an airport design. To begin, draw four runways and/or taxi lanes on a large piece of paper.

- They must be parallel or perpendicular to each other.
- Runways are 1200 m to 1500 m long, and 30 m to 60 m wide.
- Taxi lanes connect the runways. They are 20 m to 30 m wide. Their lengths can be whatever works in your design.
- Draw so that 1 cm represents 100 m.

Study the diagram for ideas. You will add to your design throughout the chapter.

If 1 cm represents 100 m, what does 1 mm represent?

WWW Web Link

To research airport designs go to www.mathlinks7.ca and follow the links.

3.2 Draw Perpendicular Bisectors

Focus on...

After this lesson, you will be able to...

- ❑ draw a line that divides a line segment in half and is at right angles to it

Many mathematical ideas are used in the designs of buildings. These math concepts make buildings more attractive to the eye and make them safer and longer lasting. Do you see any perpendicular bisectors in this photo of the Hellenic Academy in Greece?

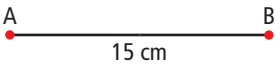
Explore the Math

Materials

- tracing paper
- ruler
- protractor

What is a perpendicular bisector?

1. Use a ruler to draw a 15-cm line segment on a piece of tracing paper. Label the endpoints A and B.



2. Fold the piece of paper so that points A and B lie on top of each other.

3. Use a ruler to draw a line segment on the crease. Label this line segment CD. Label the point where the two line segments intersect P.

4. Use a ruler to measure lengths AP and BP. What do you notice?

5. Use a protractor to measure the four angles made by the intersecting line segments. What do you notice about these angles?

Literacy Link

Bisect

Bi means "two."
Sect means "cut."
So, to bisect means to cut in two.

Reflect on Your Findings

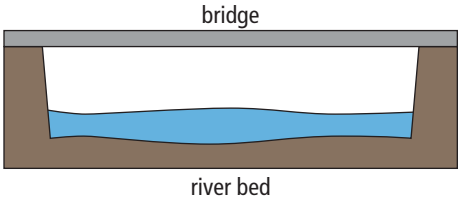
6. What can you conclude about

a) the lengths of the two parts of a bisected line segment?

b) the measure of the angles made by a perpendicular bisector?

Example: Draw a Perpendicular Bisector

A bridge over a river needs a perpendicular support under it. Draw the support in the middle of the bridge.



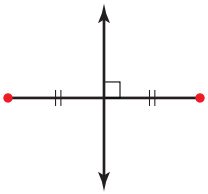
Solution

Method 1: Use a Compass

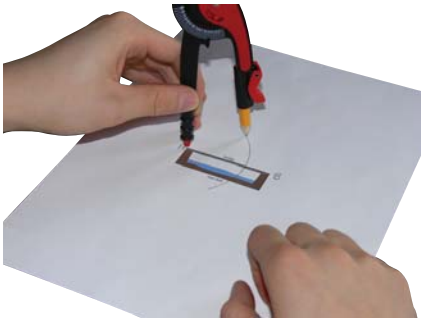
Choose a point at each end of the bridge. Label one point A and the other B.

perpendicular bisector

- a line that divides a line segment in half and is at right angles to it
- equal line segments are marked with "hash" marks



To draw the **perpendicular bisector** to AB, open your compass to a distance greater than half of AB. Place the compass point on A and draw an arc.



Keep the compass opening the same for both arcs.

Place the compass point on B and draw a second arc. Label the points of intersection C and D.

Use a ruler to draw a line segment from C to D. CD is the perpendicular bisector of AB. Highlight the part of line segment CD from the bridge to the river bed. This is the support.

Method 2: Use a Right Triangle

Choose a point at each end of the bridge. Label them A and B. To draw the perpendicular bisector, use a ruler to measure the length from point A to point B. Divide this length in half. Locate and label the middle point C.

For the right triangle, use a plastic triangle from a math set or cut a right triangle from the corner of a piece of paper.

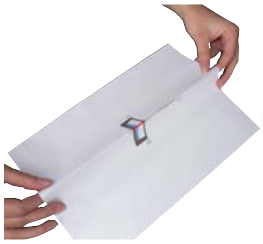
Use a right triangle to draw a line segment at point C perpendicular to AB. Extend this perpendicular bisector so that it touches the river bed. The part of the line segment from the bridge to the river bed is the support.

Remember to mark equal line segments with hash marks.

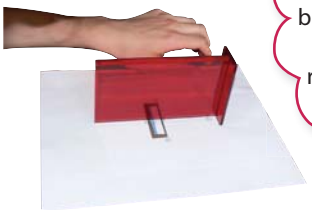
Check:

Verify a perpendicular bisector using one of these methods:

- Use paper folding.
- Use a Mira.



Fold along perpendicular bisector CD. Points A and B should lie on top of each other.



Place the Mira on perpendicular bisector CD. Points A and B should reflect onto each other.

Show You Know

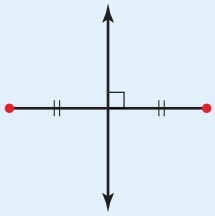
Draw a 12-cm line segment. Label it PQ. Draw the perpendicular bisector of this line segment. Label the point where they intersect R.

a) What are the lengths of PR and RQ?

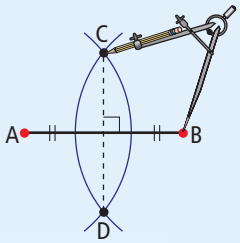
b) What does it mean if the lengths are different?

Key Ideas

- A perpendicular bisector is a line that divides a line segment in half and is at right angles (90°) to the line segment.



- Some ways to create a perpendicular bisector include using a compass, using a ruler and a right triangle, and using paper folding.



Communicate the Ideas

1. What is a perpendicular bisector?

2. What are two examples of perpendicular bisectors in the real world? Sketch each example. Share your examples with a classmate.

3. How would you explain to a friend that the post of the totem pole is a perpendicular bisector of the wings?

Practise

For help with #4 and #5, refer to the Example on page 90–91.

4. Copy each of these line segments. Draw the perpendicular bisectors. Verify the perpendicular bisectors using a method of your choice.

a) A B

b) C D

5. Copy the line segments below. Draw the perpendicular bisector of each of them. Verify the perpendicular bisectors using a method of your choice.

a) K L

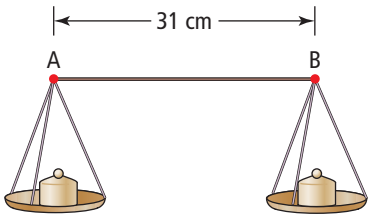
b) P Q

Apply

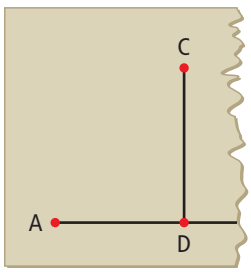
6. **a)** Draw a rectangle with a length of 8 cm and a width of 5 cm. Draw the perpendicular bisector of each side.

b) Where do these lines intersect?

7. This pan balance is missing its perpendicular bisector support. Copy the diagram and draw where the support should be. Label the top point of the support C and the bottom point D. Label the length measures of AC and BC. Label the angle measures of ∠ACD and ∠BCD.

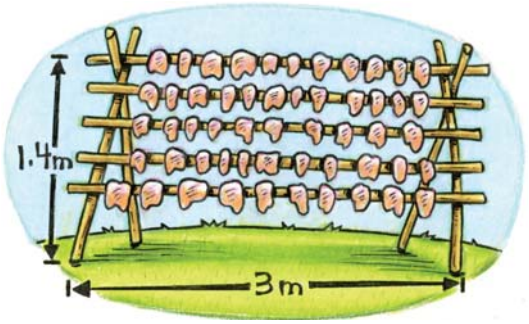


8. CD is the perpendicular bisector of line segment AB. Point B has been torn away. Copy the diagram and show two ways to replace B.

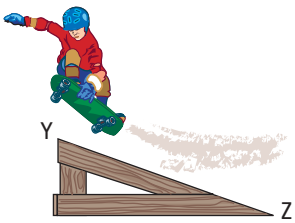


Extend

9. In some First Nations communities, fish are dried on a drying rack like the one shown. An extra support is needed for this drying rack to hold all the salmon that were caught. Use what you know about drawing perpendicular bisectors to explain how to do this. Include the lengths shown in the picture in your explanation.



10. Brian has built a ramp for the skateboard park. To make the ramp safer he decides to add a support that is the perpendicular bisector of YZ. Draw the ramp. Draw the support so that it extends from the top edge of the ramp to the bottom edge. What is the length of the support to the nearest hundredth of a metre?



1 cm represents 0.5 m

MATH LINK

Add one runway or taxi lane to your airport design that is a perpendicular bisector. Explain why you chose that location for the new runway or taxi lane.

WWW Web Link

For information about runways, go to www.mathlinks7.ca and follow the links.

3.3 Draw Angle Bisectors

Focus on...

After this lesson, you will be able to...

- ❑ draw lines that divide angles in half

Carpenters work with wood. One job a carpenter does is install wood mouldings. To place mouldings in a corner such as the one shown in this roof peak, a carpenter must first measure the angle of the corner. The next task is to cut the mouldings at an angle so that the two pieces fit together tightly. The carpenter is creating an angle bisector of the corner angle.

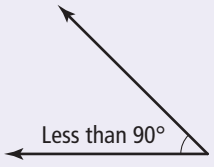
Materials

- tracing paper
- ruler
- protractor

Literacy Link

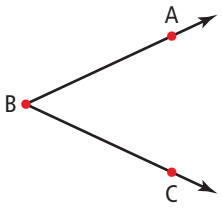
Acute Angle

An angle that is less than 90° is called an acute angle.



Explore the Math

What is an angle bisector?



1. Use a ruler to draw an acute angle on a piece of tracing paper as shown. Label the angle ABC.

2. Fold the piece of paper at B so that AB and BC lie on top of each other.

3. Use a ruler to draw a line segment on the crease you just created. Label this line segment BD.

Reflect on Your Findings

4. Use a protractor to measure ∠ABD and ∠DBC.

a) What do you notice about the measures of these two angles?

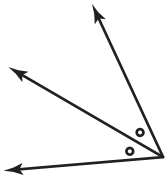
b) What can you conclude about an angle bisector?

Example: Draw an Angle Bisector

You own a sign company. You need to paint an exit sign with an arrow for a client. You have drawn the arrowhead. Draw the **angle bisector** of the arrowhead to complete the arrow.

angle bisector

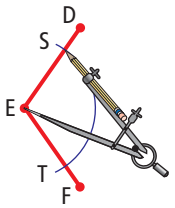
- the line that divides an angle into two equal parts
- equal angles are marked with the same symbol



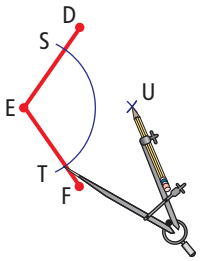
Solution

Method 1: Use a Compass

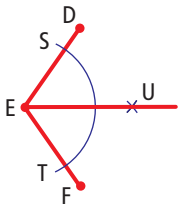
Draw and label the angle DEF. Place your compass point on E. Draw an arc as shown. Label the points of intersection S and T.



Place your compass point on S and draw an arc. Then place the compass point on T and draw an arc. Label the point of intersection U.



Use a ruler to draw a line segment from point E to point U. The angle bisector of ∠DEF is EU. Extend EU to make an arc.

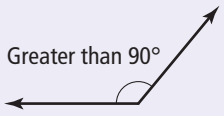


Keep the size of the compass opening the same as you draw these two arcs.

Literacy Link

Obtuse Angle

An angle that is greater than 90° and less than 180° is called an obtuse angle.

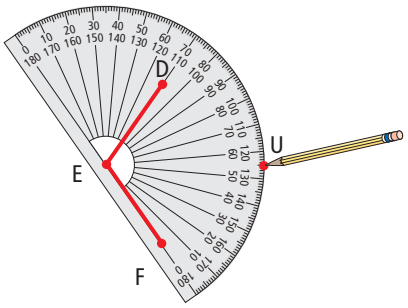


Method 2: Use a Protractor

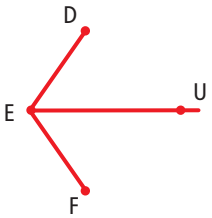
Draw and label the angle DEF.
Measure ∠DEF.
Divide its angle measure of 110° in half.

$110 \div 2 = 55$

Use a protractor to mark 55°.
Label this point U.



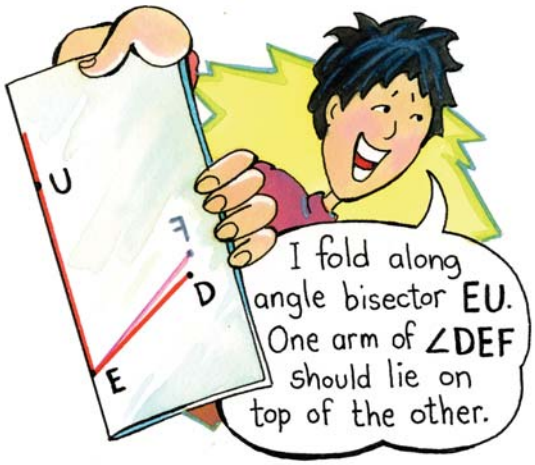
Use a ruler to draw a line segment from point E to point U. The angle bisector of ∠DEF is EU. Extend EU to make an arrow.

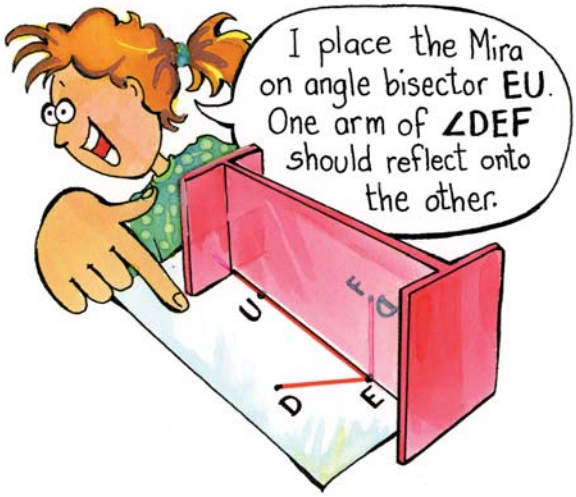


Check:

Verify an angle bisector using one of these methods:

- Use paper folding.
- Use a Mira.





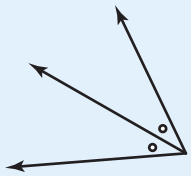
Show You Know

∠ABC is 68°. Draw the angle bisector of ∠ABC using two methods.

Remember to mark equal angles with the same symbol.

Key Ideas

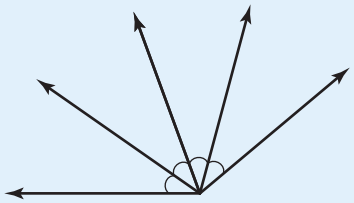
- An angle bisector divides an angle into two equal parts.
- Some ways to create an angle bisector include
  – using a ruler and a compass
  – using a ruler and a protractor
  – using paper folding



Communicate the Ideas

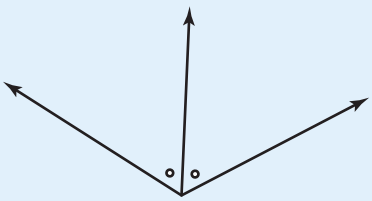
1. What is an angle bisector?

2. How could you divide an angle into four equal angles as shown?

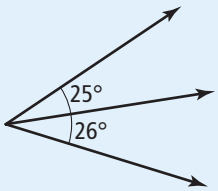


3. Has each angle been bisected? How do you know?

a)



b)



4. What are two examples of angle bisectors in the real world? Sketch each example. Share your examples with a partner.

5. Draw an obtuse angle and label it PQR.

a) Use paper folding to bisect ∠PQR. Describe what you did.

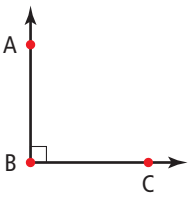
b) Label the angle bisector. Name the two angles with the same measure.

Practise

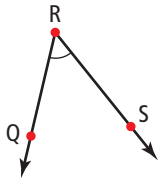
For help with #6 and #7, refer to the Example on page 95–96.

6. Draw each of the angles shown. Then, draw the angle bisectors.

a)

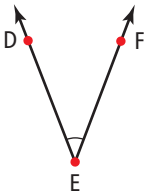


b)

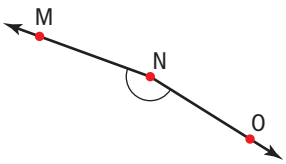


7. Copy the angles shown and then draw the angle bisectors.

a)

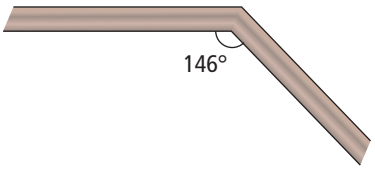


b)



Apply

8. A carpenter measures this angle of a baseboard to be 146°.



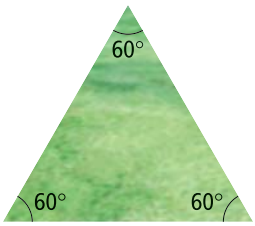
a) Draw the angle. Then, draw the angle bisector.

b) Verify the angle bisector using a method of your choice.

c) Measure and record the two angles made by the angle bisector.

9. Draw an 88° angle and bisect it. Predict the measure of the smallest angles if you bisect one of the new angles. Bisect one of the new angles. Verify your prediction by measuring.

10. Suki is making a dinosaur costume for her little brother. She has cut out a triangle of fabric as shown. She wants to cut the triangle into two equal pieces. Using what you know about angle bisectors, show how she can do this. Use two different methods.

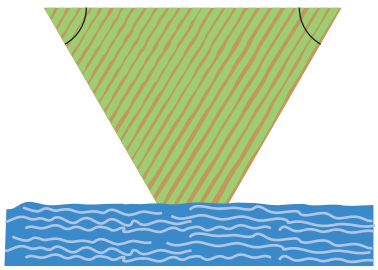


11. a) Draw a square using a protractor and ruler.

b) Bisect the four angles of the square.

c) What two things do you notice about the bisectors?

12. A family lives on a farm that is the shape shown. The parents wish to divide the land equally between their son and daughter. They want each child to have some land that borders the river. How can the parents use an angle bisector to do this? Explain.



Extend

13. Ilana built a ramp for the skateboard park. She decides to add a support that is an angle bisector of vertex X. Draw the ramp. Draw the new support so that it extends from X to the top edge of the ramp. What is the length of the support in metres?

14. An artist wants to draw a sun with eight light rays coming out of it. The rays will be spaced apart equally. Show how this can be done using perpendicular bisectors and angle bisectors.

15. Draw an acute triangle with angles of 58°, 46°, and 76°.

a) Draw the angle bisector for each angle. What do you notice about the three angle bisectors?

b) Place your compass point at the point of intersection of the three angle bisectors. Construct a circle so it fits just inside the triangle. What do you notice about where the circle touches the triangle?

MATH LINK

Add another runway or taxi lane to your airport design that bisects the angle made by two others. Study the diagram of the Calgary International Airport as an example. Explain why you chose that location for your runway or taxi lane.

WWW Web Link

For practise finding the best location for airport runways, go to www.mathlinks7.ca and follow the links.

3.4 Area of a Parallelogram

Focus on...

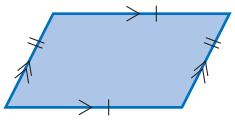
After this lesson, you will be able to...

- develop the formula for the area of a parallelogram
- calculate the area of a parallelogram

One of the shapes a marching band can make is a rectangle. Another is a **parallelogram**. How do you calculate the area of the rectangle? Can you use the same method to calculate the area of the parallelogram?

parallelogram

- a four-sided figure with opposite sides parallel and equal in length



Explore the Math

How do you determine the area of a parallelogram?

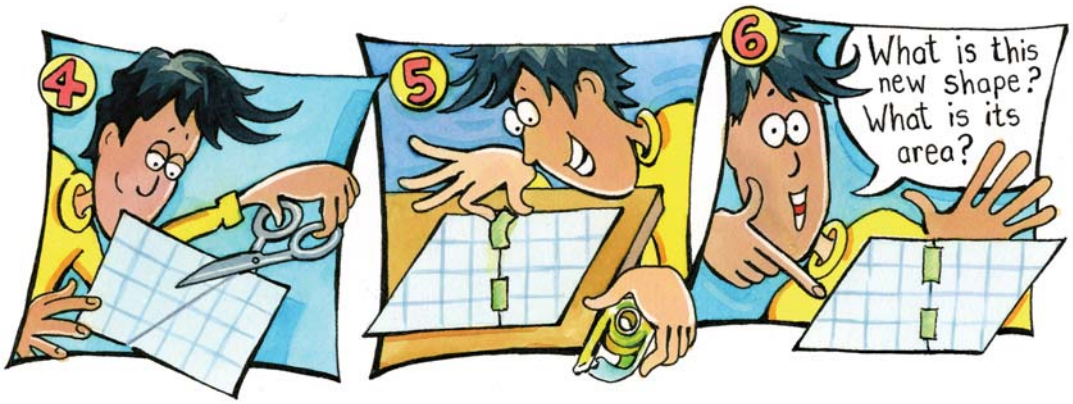
Materials

- centimetre grid paper
- ruler
- scissors
- tape

1. On centimetre grid paper, draw a rectangle that is 6 cm long and 4 cm wide. Cut out the rectangle with scissors.

2. Count the number of square centimetres the rectangle covers. What is the area of this rectangle?

3. Use scissors to cut across the rectangle as shown. Tape the two pieces together.



4. What shape did you form? What do you know about this shape that helped you to identify it?

5. Is the area of the parallelogram the same as that of the original rectangle? How do you know?

6. **a)** Predict the length of the **base** (b) of the parallelogram. Verify by measuring with a ruler.

b) Predict the **height** (h) of the parallelogram. Verify by measuring with a ruler.

c) Is h parallel or perpendicular to b of the parallelogram?

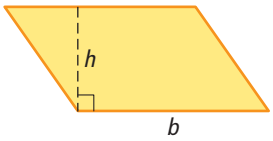
7. What is the relationship between b and h, and the area of the parallelogram?

base

- a side of a two-dimensional closed figure
- common symbol is b

height

- the perpendicular distance from the base to the opposite side
- common symbol is h



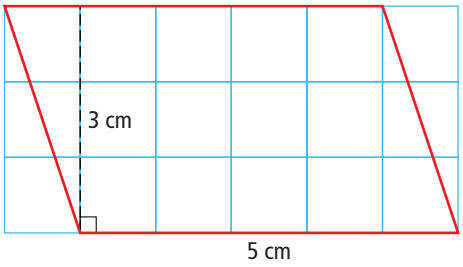
Reflect on Your Findings

8. **a)** Suggest a formula for calculating the area of a parallelogram.

b) Compare your formula with those of your classmates. Discuss any differences and make sure that everyone agrees on the formula.

Example 1: Determine the Area of a Parallelogram

The parallelogram shown has a base of 5 cm and a height of 3 cm. Use a formula to determine its area.



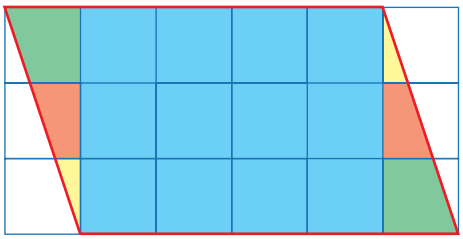
Solution

Base (b) is 5 cm and height (h) is 3 cm. Substitute the values into the formula for the area of a parallelogram.

A = base × height
$A = b \times h$
$A = 5 \times 3$
$A = 15$

The area of the parallelogram is 15 cm^2.

Check:
Verify your answer. Estimate the area by counting squares.



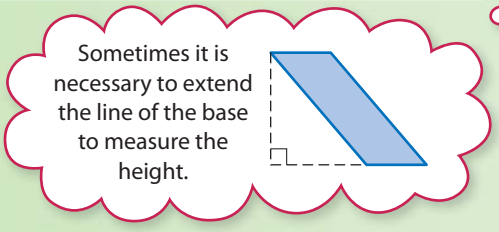
Count full squares: 12 squares.
Count squares that are almost full: 2 squares.
Count squares that are about half full: 2 half squares = 1 full square.
Do not count almost empty squares.

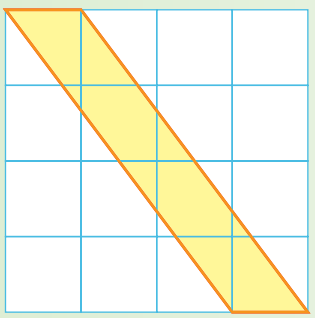
$12 + 2 + 1 = 15$

The estimate for the area of the parallelogram is 15 cm^2.

Show You Know

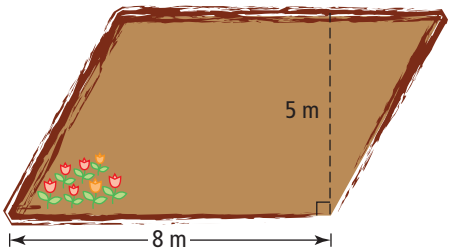
This parallelogram was drawn on centimetre grid paper. Use a formula to determine its area. Check your answer using estimation.





Example 2: Make Calculations Using the Area of a Parallelogram

Jessica has created a tulip garden in the shape of a parallelogram. She is going to plant 10 tulips per square metre of garden. Jessica's garden has a base of 8 m and a height of 5 m. How many tulips does Jessica need?



Solution

Understand

- What is the area of the garden?
- How many tulips does Jessica need?

Plan

1. Determine the area of the garden.

2. Determine the number of tulips needed for the area.

Do It!

1. To find the area of the garden, substitute the values into the formula.

$A = b \times h$

$A = 8 \times 5$

$A = 40$

The area of the garden is 40 m^2.

2. Jessica is going to plant 10 tulips per square metre. That means each square metre of her garden will have 10 tulips in it.

1 m^2 → 10 tulips

2 m^2 → 20 tulips

3 m^2 → 30 tulips

The pattern is to multiply the number of square metres by 10 to get the number of tulips.

$40 \times 10 = 400$

Jessica needs 400 tulips.

Strategies

Look for a Pattern

Refer to page xvii.

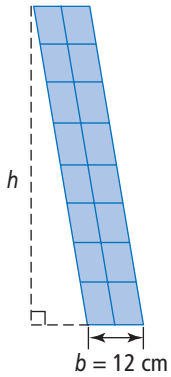
Look Back

Check:

Work backward by dividing 400 tulips by 10. The answer should be the area of Jessica's garden. $400 \div 10 = 40$. The calculated area for Jessica's garden was 40 m^2.

Example 3: Determine the Height of a Parallelogram

Diana has enough parallelogram-shaped tiles to cover a section of wall with an area of 840 cm^2. The design will have a base of 12 cm. How high can she make her design?



Solution

Determine the height (h). The base b is 12 cm and the area is 840 cm^2. Substitute these values into the formula for the area of a parallelogram.

$A = b \times h$

$840 = 12 \times h$

Guess and check to determine h.

Try $h = 50$ $\quad 12 \times 50 = 600$ Too low.

Try $h = 80$ $\quad 12 \times 80 = 960$ Too high.

Try $h = 70$ $\quad 12 \times 70 = 840$ Correct!

Diana can make the height of the design 70 cm.

Strategies

Guess and Check

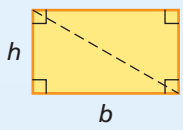
Refer to page xvi.

Show You Know

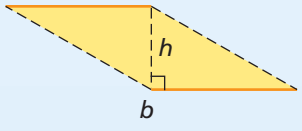
A parallelogram has a base of 90 m and an area of 450 m^2. Determine the height of the parallelogram.

Key Ideas

- The formula for the area of a rectangle can be used to determine the formula for the area of a parallelogram.
- The formula for the area of a parallelogram is $A = b \times h$, where b is the base and h is the height.
- The height of a parallelogram is always perpendicular to its base.



$A = b \times h$



$A = b \times h$

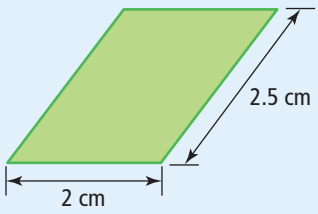
Communicate the Ideas

1. Explain why the area of a rectangle and the area of a parallelogram made from the rectangle are the same. Include diagrams with your answer.

2. Deepa determined the area of this parallelogram to be 5 cm^2.

a) Explain her error.

b) Do the calculation correctly.



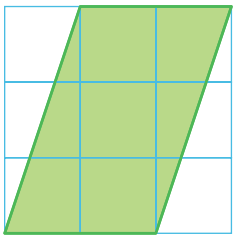
Practise

For help with #3 to #6, refer to Example 1 on page 102.

3. These parallelograms are drawn on centimetre grid paper. What is the area of each parallelogram?

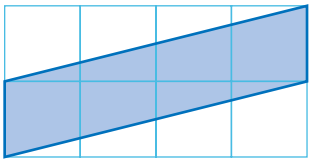
a)

b)

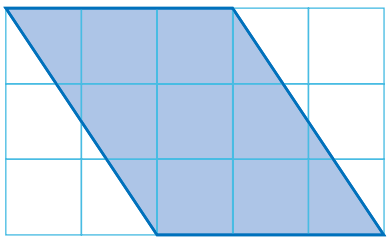


4. Determine the area of each parallelogram. They are drawn on centimetre grid paper.

a)



b)



5. Draw each of the following parallelograms on centimetre grid paper. Use the formula to determine the area of each parallelogram. Check your answers using estimation.

a) $b = 4$ cm, $h = 5$ cm

b) $b = 3$ cm, $h = 7$ cm

6. Draw each parallelogram on centimetre grid paper. Determine each area using the formula. Check your answers using estimation.

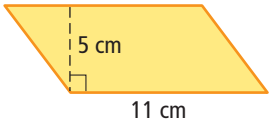
a) $h = 6$ cm, $b = 4$ cm

b) $h = 2$ cm, $b = 4$ cm

For help with #7 and #8, refer to Example 2 on page 103.

7. What is the area of each parallelogram?

a)

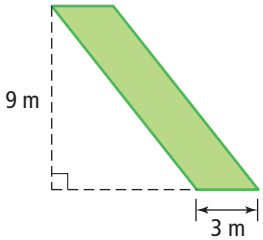


b) $b = 7$ cm, $h = 9$ cm

c) The height is 6.2 m and the base is 3 m.

8. Determine the area of each parallelogram.

a)

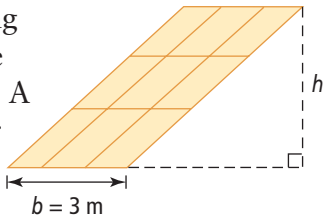


b) $b = 9.2$ mm, $h = 12.3$ mm

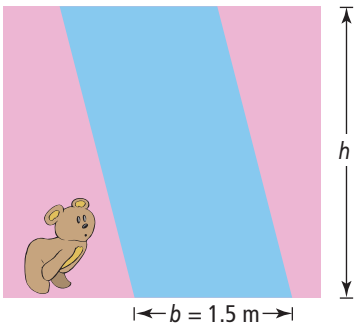
c) The base is 4.5 cm and the height is twice the base.

For help with #9 and #10, refer to Example 3 on page 104.

9. Workers are putting new flooring in the community centre. A section of the floor is going to have a parallelogram-shaped design. There are enough floor tiles to cover an area of 12 m^2. The base will be 3 m. What can the height of the design be?

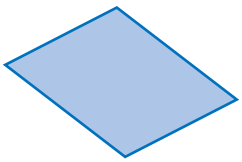


10. A parallelogram-shaped stripe is going to be painted on the wall of a daycare centre. There is enough paint to make a stripe with an area of 9 m^2. How high can the stripe be?



Apply

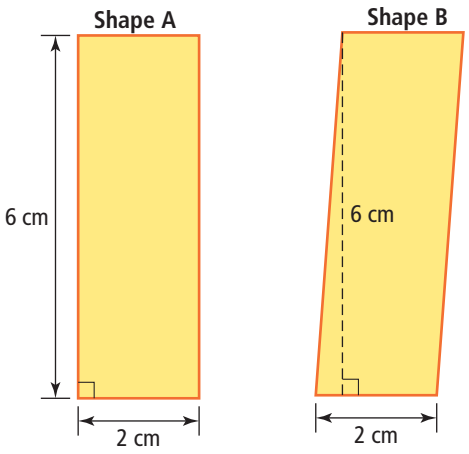
11. a) With a ruler, measure the base and height of this parallelogram. Explain how you measured.



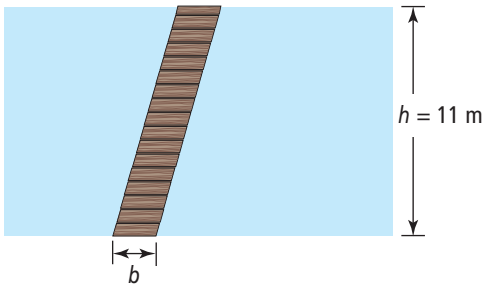
b) What is the area?

c) Use a different side of the parallelogram as the base. Determine the new height and calculate the area. How does this compare to your answer in b)?

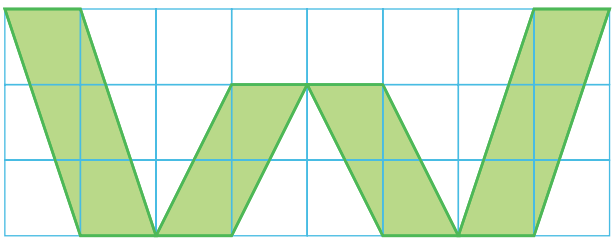
12. Which of the two shapes shown below has a greater area? Explain your answer.



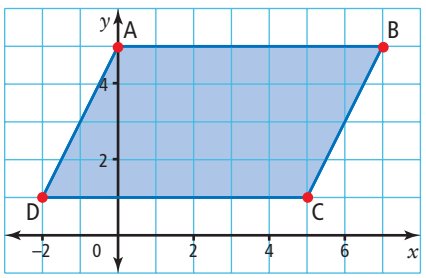
13. The spring thaw has caused a rectangular section of the schoolyard to flood. A parallelogram-shaped wooden walkway is being placed over it as shown. The area of the walkway is 22 m^2. How wide will the base of the walkway be in metres?



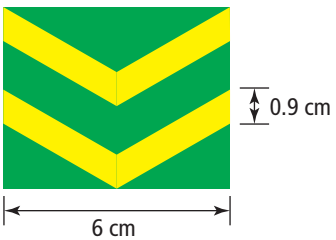
14. Wanda has designed a logo of her first initial on centimetre grid paper. She wants to find the area of the logo so she can get fabric to sew it on her jacket. What is the area of the initial?



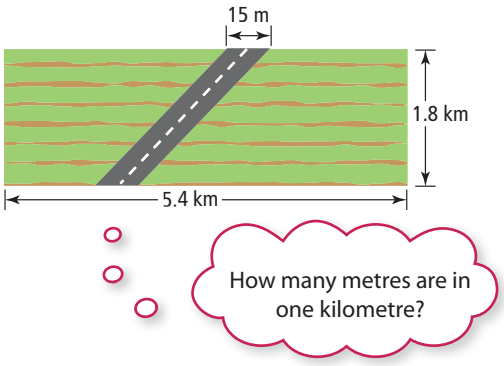
15. What is the area of parallelogram ABCD?



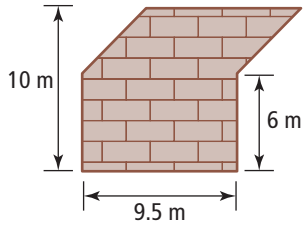
16. What is the area of the yellow portion of this corporal's insignia? Use a calculator to determine the answer.



17. A construction company is building a road through a rectangular section of farmland. The farmer wants to know the area of land being used for the road. Determine the area in square metres.



18. a) What is the area of paving stone needed for the patio?

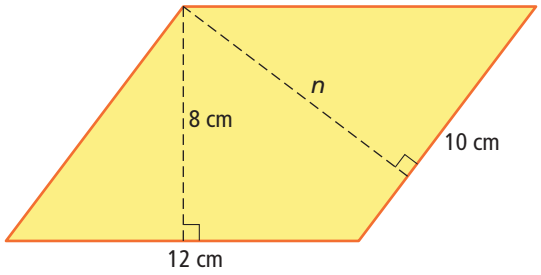


b) Paving stones cost $55 per square metre. Estimate how much it would cost to pave the patio. Show your work.

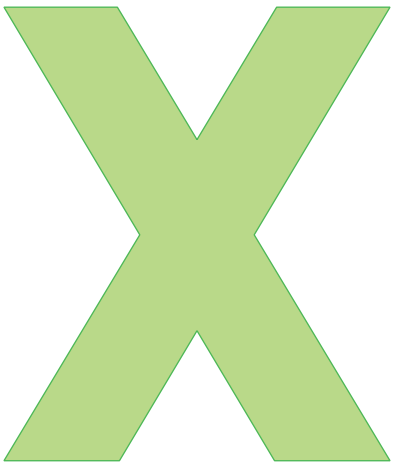
c) Calculate the cost.

Extend

19. The figure below is a parallelogram. What is the length of n?



20. What is the area of this X? Explain how you calculated your answer.



MATH LINK

a) You are going to include in your airport design an area that is in the shape of a parallelogram. Add as many runways and taxi lanes as necessary to create a parallelogram.

b) Determine the area of the parallelogram. If you have more than one parallelogram, use the largest one.

c) You decide to plant grass in the parallelogram-shaped area. If one bag of grass seed covers 1000 m^2, how many bags of seed will you need for the parallelogram?

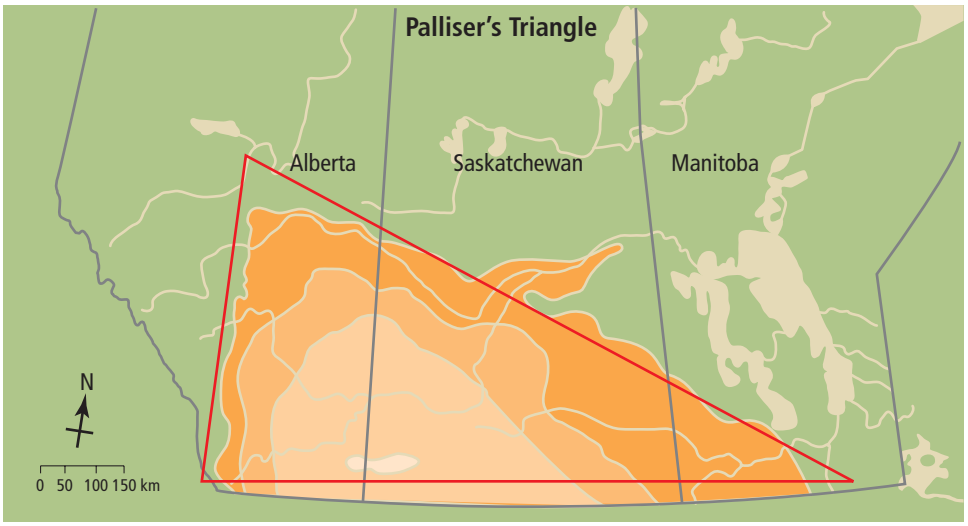
3.5

Area of a Triangle

Focus on...

After this lesson, you will be able to...

- ❑ develop the formula for the area of a triangle
- ❑ calculate the area of a triangle



Palliser's Triangle is the driest region of the Canadian prairies. It stretches from southwestern Manitoba, through Saskatchewan, to southern Alberta. Despite its dry climate, over half of Canada's agricultural production takes place in this region. How could you determine the area of Palliser's Triangle?

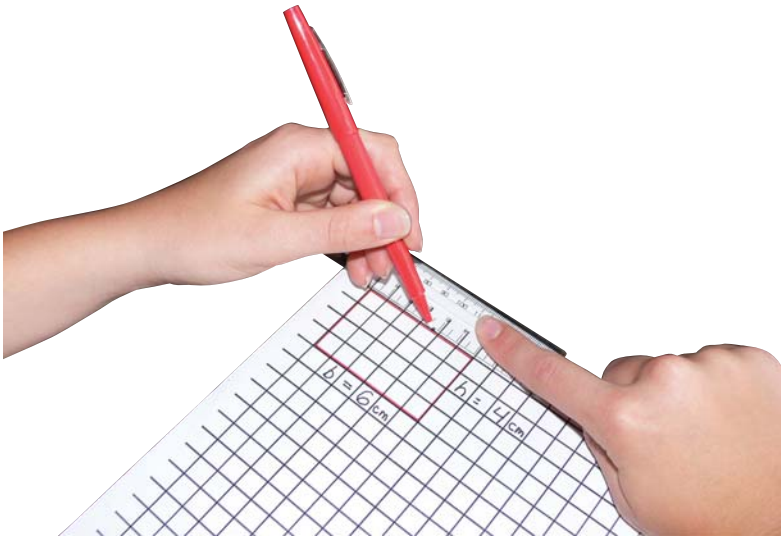
Explore the Math

Materials

- centimetre grid paper
- ruler
- scissors

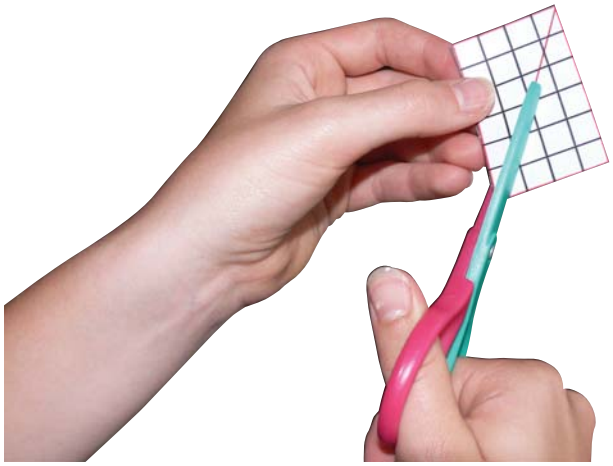
How can you determine the area of a triangle?

1. On centimetre grid paper, draw a rectangle that has a base of 6 cm and a height of 4 cm. Cut out the rectangle with scissors.



2. Count the number of square centimetres. What is the area of this rectangle?

3. Draw a diagonal line from one corner of the rectangle to the opposite corner. Use scissors to cut along the diagonal line as shown.



4. What are the two new shapes you created?

5. Place these two shapes on top of each other. How do they compare?

6. What is the relationship between the area of one of the triangles and the area of the original rectangle? Explain how you know.

7. a) Predict the length of the base (b) of each triangle. Use a ruler to measure it.

b) Predict the height (h) of each triangle. Use a ruler to measure it.

8. Is h parallel or perpendicular to b of the triangle?

9. What is the relationship between b and h, and the area of the triangle?

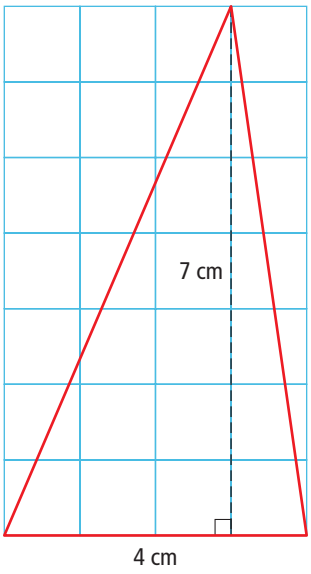
Reflect on Your Findings

10. a) Suggest a formula for calculating the area of a triangle.

b) Compare your formula with those of your classmates. Discuss any differences and make sure that everyone agrees on the formula.

Example 1: Determine the Area of a Triangle on a Grid

This triangle has a base of 4 cm and a height of 7 cm. Determine its area using a formula.



Solution

Base (b) is 4 cm and height (h) is 7 cm.
Substitute the values into the formula for the area of a triangle.

$A = \text{base} \times \text{height} \div 2$
$A = b \times h \div 2$
$A = 4 \times 7 \div 2$
$A = 28 \div 2$
$A = 14$

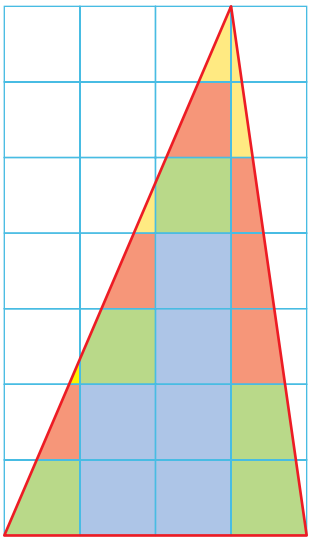
The area of the triangle is 14 cm^2.

Check:
Verify your answer. Estimate the area by counting squares.

Count full squares: 6 squares.
Count squares that are almost full: 5 squares.
Count squares that are about half full:
6 half squares = 3 full squares.
Do not count almost empty squares.

$6 + 5 + 3 = 14$

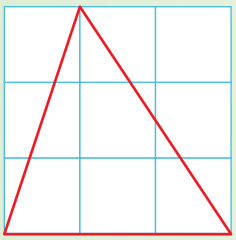
The estimate for the area of the triangle is 14 cm^2.



You can also count this way: a yellow part and a green part make a full square.

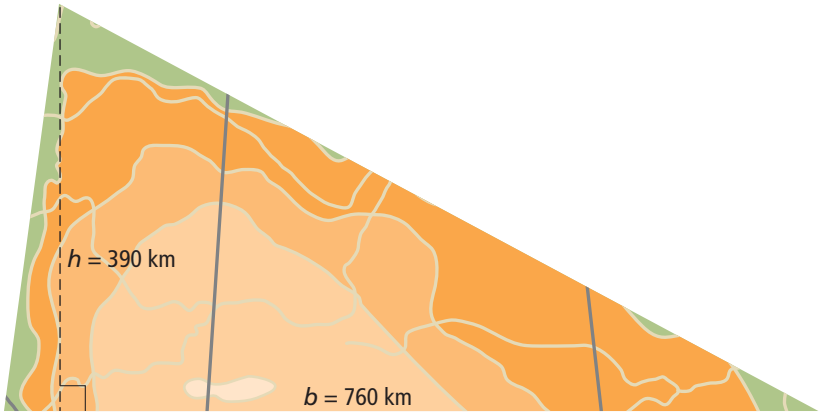
Show You Know

The triangle shown has been drawn on centimetre grid paper. Use a formula to determine its area. Check your answer using estimation.



Example 2: Determine the Area of a Triangle

Palliser's Triangle has a base of approximately 760 km and a height of approximately 390 km. What is the area of Palliser's Triangle?



Solution

$b = 760$ km
$h = 390$ km

Substitute the values into the formula for the area of a triangle.

$$A = \frac{b \times h}{2}$$

$$A = \frac{760 \times 390}{2}$$ Substitute the values.

$$A = \frac{296\,400}{2}$$

$$A = 148\,200$$

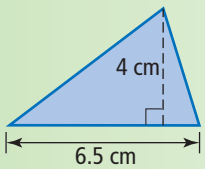
The area of Palliser's Triangle is 148 200 km^2.

Literacy Link

There is more than one way to write the formula for the area of a triangle:
$\frac{b \times h}{2}$ means the same as $b \times h \div 2$.

Show You Know

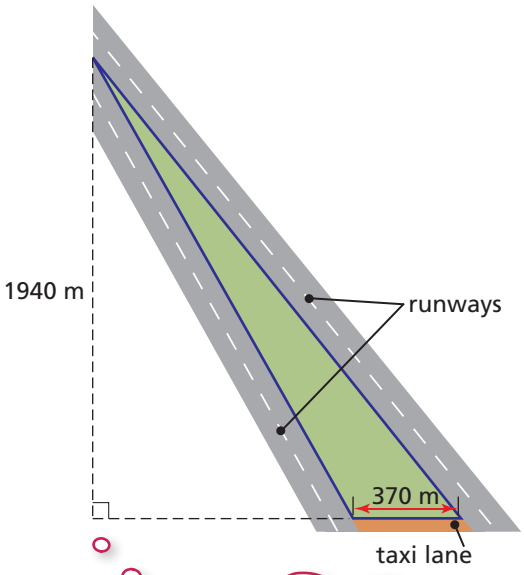
What is the area of the triangle?



Example 3: Make Calculations Using the Area of a Triangle

An airport has two new runways and a taxi lane that connects them as shown.

The area within the runways and taxi lane is going to be seeded with grass. It is necessary to use 6 g of seed per square metre. How much seed is needed for the triangular area? Give your answer in kilograms.



Sometimes it is necessary to extend the line of the base to measure the height.

Solution

$b = 370$ m, $h = 1940$ m

$A = b \times h \div 2$

$A = 370 \times 1940 \div 2$ Substitute the values.

$A = 717\,800 \div 2$

$A = 358\,900$

The area to be seeded is 358 900 m^2.

Strategies

Look for a Pattern

Refer to page xvii.

For each square metre of the triangular area, 6 g of grass seed is needed.

1 m^2 → 6 g of grass seed
2 m^2 → 12 g of grass seed
3 m^2 → 18 g of grass seed

The pattern is to multiply the number of square metres by 6.

$358\,900 \times 6 = 2\,153\,400$

$2\,153\,400$ g $= 2153.4$ kg

2154 kg of grass seed are needed.

The answer in kilograms is 2154 kg, not 2153 kg, because it is okay to have a bit of extra seed, but not to have too little seed.

Literacy Link

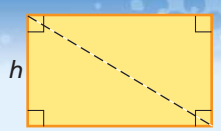
To convert 1500 g to kilograms, divide by 1000.

$1500 = \frac{1500}{1000}$

1500 g = 1.5 kg

Key Ideas

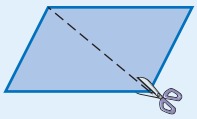
- The formula for the area of a rectangle or parallelogram can be used to determine the formula for the area of a triangle.



- The formula for the area of a triangle is $A = b \times h \div 2$ or $A = \frac{b \times h}{2}$, where b is the base of the triangle and h is the height of the triangle.
- The height of a triangle is always perpendicular to its base.

Communicate the Ideas

1. You cut a parallelogram along the line joining opposite vertices as shown. Now, you have two triangles. What is the relationship between the area of the parallelogram and the area of one triangle?



2. a) Draw a triangle to match this solution:

$A = b \times h \div 2$
$A = 12 \times 8 \div 2$
$A = 48$

The area is 48 cm^2.

b) Exchange with a partner. Did you draw the same triangle? Are different triangles possible? Discuss with your partner.

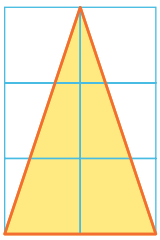
3. Sofia calculated the area of a triangle with a base of 7 cm and a height of 5 cm to be 17.5 cm. What mistake did Sofia make? Explain.

Practise

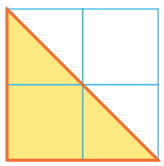
For help with #4 to #7, refer to Example 1 on page 110.

4. These triangles are drawn on centimetre grid paper. What is the area of each triangle?

a)

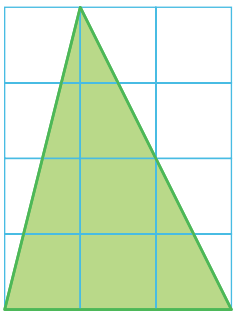


b)

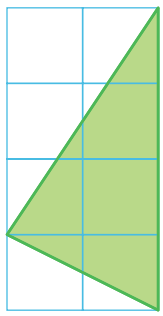


5. Determine the area of each triangle drawn on centimetre grid paper.

a)



b)



6. Draw each triangle on centimetre grid paper. Determine the area of each triangle, using the formula. Check your answers using estimation.

a) $b = 6$ cm, $h = 7$ cm

b) $b = 5$ cm, $h = 4$ cm

7. Draw each triangle on centimetre grid paper. Use the formula to determine the area of each triangle. Check your answers using estimation.

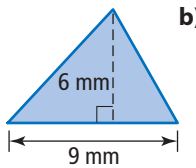
a) $h = 2$ cm, $b = 8$ cm

b) $h = 5$ cm, $b = 3$ cm

For help with #8 and #9, refer to Example 2 on page 111.

8. Determine the area of each triangle.

a)

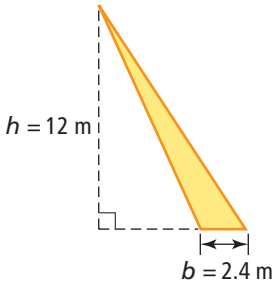


b) $b = 8$ cm, $h = 10$ cm

c) The base is 11.8 mm and the height is 14.6 mm.

9. What is the area of each triangle?

a)

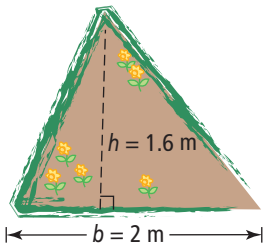


b) The base is 8.8 cm and the height is half the base.

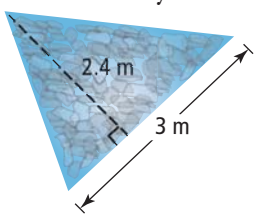
c) $h = 2.7$ m, $b = 40$ cm

For help with #10 and #11, refer to Example 3 on page 112.

10. Daniel wants to plant as many daffodils as he can in a triangular garden. He finds out that he should plant no more than 49 daffodils for each square metre of garden. How many daffodils can he plant in his garden?

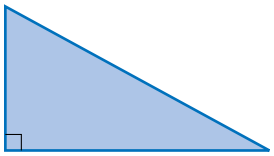


11. Workers at the botanical gardens are building a new triangular pond as shown. They need to fill the bottom with pebbles. One bag of pebbles fills 1 m^2. How many bags of pebbles will they need?



Apply

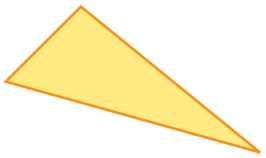
12. a) Measure the base and height of the triangle shown.



b) Explain how you measured the base and height.

c) What is the area of the triangle?

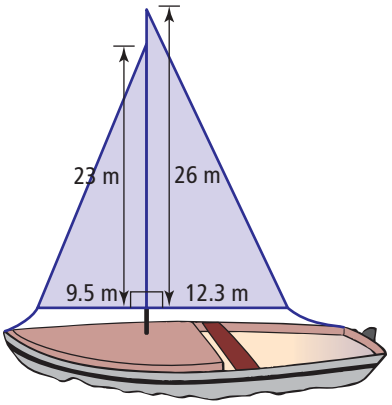
13. a) Measure the base and height of the triangle shown. Explain how you measured.



b) What is the area of the triangle?

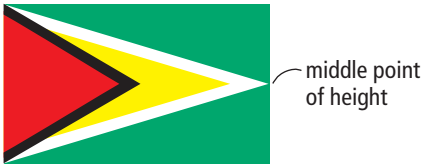
c) Use a different side of the triangle as the base. Measure the height and calculate the area. How does this area compare to the area you calculated in b)?

14. a) How much material is needed to make the two sails on the sailboat shown?



b) Would you need any additional material for the sails? Explain.

15. Nageen's parents are from Guyana. As a present for them she wants to make a flag of that country like the one shown. Nageen plans to make the flag 1 m wide and 60 cm high. How much green material will Nageen need to purchase? Do not count the extra material needed to sew the flag together. Give your answer in square metres.

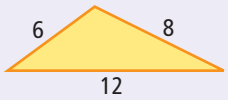


Extend

16. Two triangles have the same area. Do they also have the same perimeter? Verify your answer. Hint: Draw several triangles with the same area. Measure their perimeters.

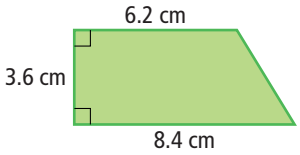
Literacy Link

To determine perimeter, find the sum of all the sides.

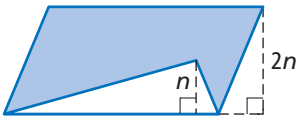


$P = 6 + 8 + 12$
$= 26$ units

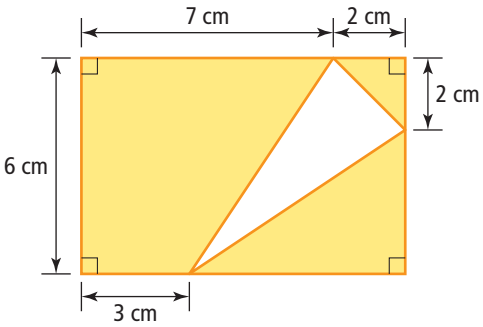
17. What is the area of this figure? Show at least two different ways to calculate the answer.



18. Emily thinks the unshaded region of the parallelogram is 25% of the parallelogram. Han thinks it is 20%. Who is correct? Explain how you know.



19. Fahad thinks the area of the white triangle is 10 cm^2. Amanda thinks it is 12 cm^2. Who is correct? Explain how you know.



MATH LINK

a) Include in your airport design an area that is in the shape of a triangle. Add as many runways and taxi lanes as necessary to create a triangle.

b) What is the area of the triangle? If you have more than one, use the largest triangle.

c) You decide to fill in the triangular area with gravel. If one truckload of gravel covers 100 m^2, how many truckloads of gravel will you need for the triangle?

3 Chapter Review

Key Words

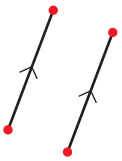
For #1 to #5, match the descriptions in column A with the words in column B.

Column A	Column B
1. Describes lines that intersect at 90°	**A** base
2. Describes lines in the same plane that never intersect	**B** angle bisector
3. A line that divides a line segment in half and is at right angles (90°) to it	**C** line segment
4. A line that divides an angle into two equal parts	**D** perpendicular
5. A part of a line between two endpoints	**E** parallel
	F perpendicular bisector

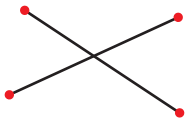
3.1 Parallel and Perpendicular Line Segments, pages 82–88

6. Which of the following are parallel and which are perpendicular?

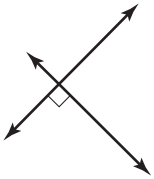
a)



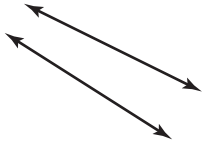
b)



c)



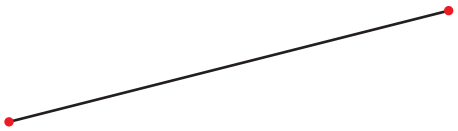
d)



7. Copy these line segments into your notebook. Draw two perpendicular line segments and two parallel line segments for each.

a)

b)

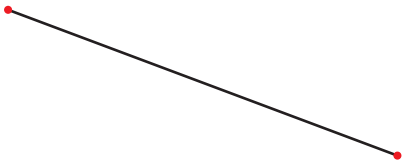


3.2 Draw Perpendicular Bisectors, pages 89–93

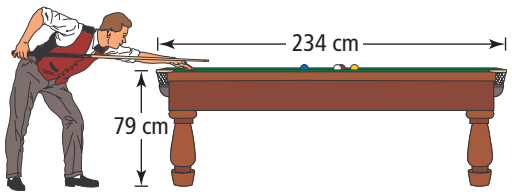
8. Copy these line segments. Draw the perpendicular bisectors using two different methods. Measure the two parts of each bisected line segment. Record each set of measurements.

a)

b)



9. A support must be placed under the pool table at its centre. Make a sketch of the pool table. Draw the position of the perpendicular support. Label the support and label the measures of the table lengths on each side of the support.



10. You draw a perpendicular bisector of a 30-cm line segment. You then draw a perpendicular bisector of one of the bisected line segments.

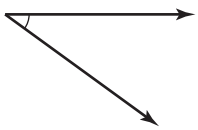
a) What is the length of the shortest line segment?

b) How does the length of the shortest line segment compare to the length of the original line segment?

3.3 Draw Angle Bisectors, pages 94–99

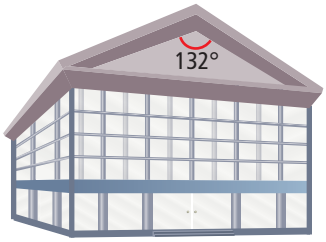
11. Draw the following angles. Draw the angle bisectors using two different methods. Measure the two angles of each bisected angle. Record each set of angle measures.

a)



b) $\angle XYZ = 120°$

12. An architect measures the angle of a roof to be 132°. Draw the angle.



a) Predict the measure of the resulting angles if you bisect the angle.

b) Bisect the angle. Verify your predictions by measuring.

13. You construct an angle bisector of an 80° angle. You then construct an angle bisector of one of the resulting angles.

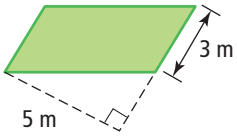
a) What is the measure of the smallest angle?

b) How does the measure of the smallest angle compare to the measure of the original angle?

3.4 Area of a Parallelogram, pages 100–107

14. What is the area of each parallelogram?

a)



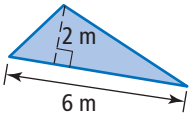
b) $b = 5.4$ mm, $h = 3.7$ mm

15. In the flag of Congo-Brazzaville, the length of the rectangle is 2.7 times the base of the yellow parallelogram. How do the areas of the rectangle and the parallelogram compare? Explain.

3.5 Area of a Triangle, pages 108–115

16. What is the area of each triangle?

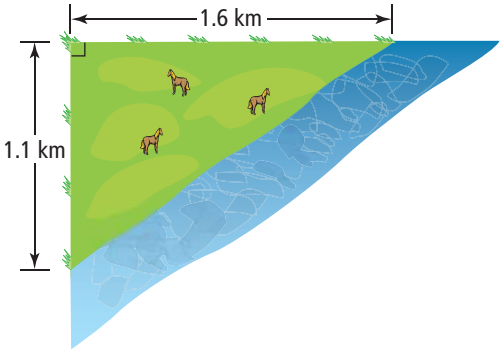
a)



b) $b = 5.2$ m, $h = 5.3$ m

17. Aleta has a pasture for her horses. The pasture runs alongside a creek as shown.

a) What is the approximate area of the pasture? Explain how you determined your answer.



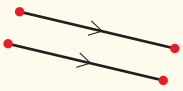
b) Why is the area approximate?

3 Practice Test

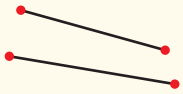
For #1 to #5, choose the best answer.

1. Which pair of line segments is parallel?

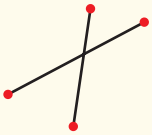
A



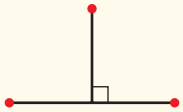
B



C

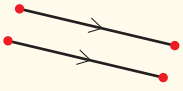


D

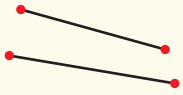


2. Which pair of line segments is perpendicular?

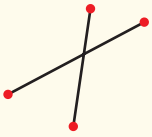
A



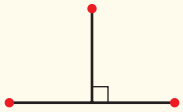
B



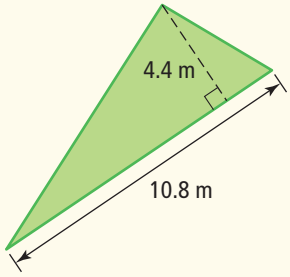
C



D

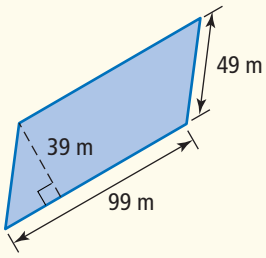


3. What is the area of this triangle, to the nearest tenth?



A 11.9 m²
B 23.8 m²
C 47.5 m²
D 95.0 m²

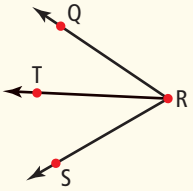
4. What is the area of this parallelogram?



A 2886 m²
B 3861 m²
C 4356 m²
D 4851 m²

5. The measure of ∠QRS is 64°. Line segment RT is its angle bisector. What is the measure of ∠QRT?

A 16°
B 32°
C 64°
D 138°



Short Answer

6. Copy each line segment. Draw a line segment parallel to each one.

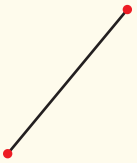
a)

b)

7. Copy each line segment. Draw a line perpendicular to each one.

a)

b)



8. Draw a line segment 12 cm long and label it AB.

a) Show two methods to construct the perpendicular bisector for line segment AB. Label the bisector FG, so that F lies on AB.

b) What is the length of line segment AF?

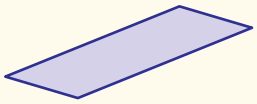
c) What is the measure of ∠AFG?

9. Draw an angle that measures 72° and label it PQR.

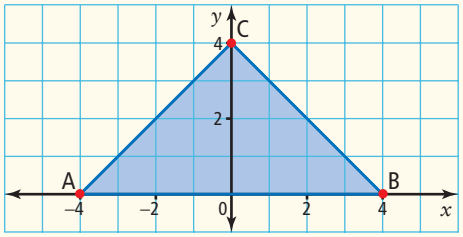
a) Show two methods to construct the angle bisector for ∠PQR. Label the angle bisector QS.

b) What is the measure of ∠PQS?

10. Use a ruler to measure the base and height of the parallelogram. What is the area?

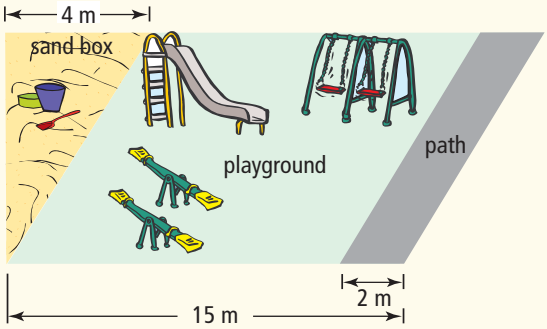


11. What is the area of △ABC?



Extended Response

12. A school playground has an area of 84.5 m².

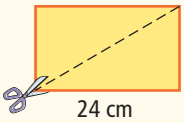


a) What are the base and height of the parallelogram-shaped playground?

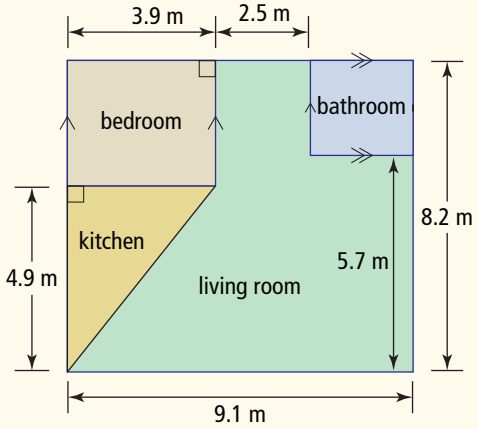
b) What is the area of the path?

c) What is the area of the sandbox?

13. Manchu has drawn a rectangle with a base of 24 cm and an area of 336 cm². He cuts the rectangle as shown to make two triangles. What are the height and area of each triangle?



14. Cheryl wants new flooring and carpeting for her rectangular apartment. A floor plan of her apartment is shown.



a) How much flooring does Cheryl need for her bathroom?

b) How much carpeting does Cheryl need for her bedroom and living room?

WRAP IT UP!

a) Complete your airport design. Make any revisions that you think will make it better. Add details such as a passenger terminal, control tower, air freight area, fire hall, aircraft maintenance area, etc. Study the diagram as an example.

b) Write a one-page report that explains why you designed your airport the way you did.

Math Games

Amazing Mazes

A maze is a type of puzzle. The solver must find a path through the maze.

Materials

- grid paper
- ruler
- tracing paper

Mazes are created in various ways. The walls can be fences, hedges, or actual solid walls. In Canada and some other countries, mazes are often cut into cornfields for fall fun.

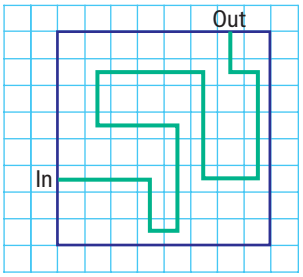
In this activity, you will design your own maze using only perpendicular and parallel walls.

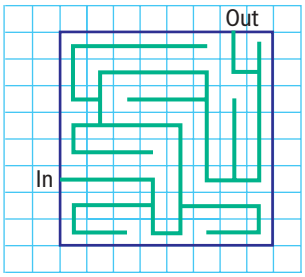
1. The following steps show how to design a maze on an 8×8 grid. You may want to practise drawing this maze on a different 8×8 grid before you design a larger one.

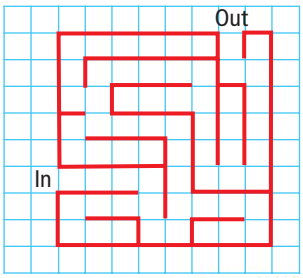
a) Mark where to enter and leave the maze. Use a ruler to draw the path from where you enter to where you leave. All parts of the path must be parallel to some grid lines and perpendicular to others.

b) Use a ruler to draw dead-end branches from your path. Stop drawing when every square on the grid includes either the path or a branch.

c) Place a sheet of tracing paper over your grid. Look at the sides of the small grid squares through the tracing paper. Use a ruler to trace only the sides that are not crossed by the path or a branch. You have now drawn the walls of the maze on the tracing paper.







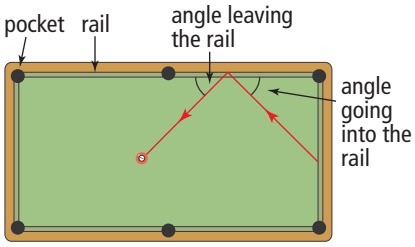
2. Design a larger maze, such as a 15×15 square or a 20×10 rectangle. Challenge a classmate to solve your maze or give copies of your maze to two classmates and see who can solve it faster.

Challenge in Real Life

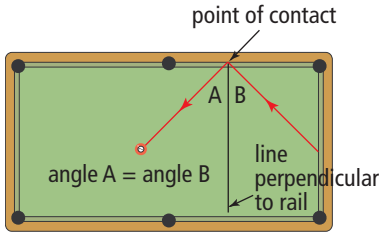
Bank Shots in the Game of Pool

In the game of pool, players use a stick, called a cue, to hit numbered balls into the pockets of a pool table.

To make a bank shot, a player bounces a ball against a rail to send it into a pocket. The angle at which a ball hits the rail is equal to the angle at which it bounces off the rail.



If you draw a line segment perpendicular to the rail at the point of contact, the two angles formed by the path of the ball are equal.



Your challenge is to make the red #7 ball go into a pocket using a bank shot. Imagine that the ball will keep rolling in straight paths and rebounding off rails until it goes into a pocket.

a) Sketch a pool table with 6 pockets. Draw the ball on the table.
b) Draw a line from the ball to any rail.
c) At the point of contact, draw a perpendicular line segment to find the angle at which the ball leaves the rail. Draw the angle and extend the line until it makes contact with another rail.
d) Keep repeating step c) until the ball rolls into a pocket. How many rebounds did it take for the ball to roll into a pocket?
e) Sketch another pool table. Place the ball at a location of your choice. Have a classmate follow steps b) to d) on your pool table. How many rebounds were there this time?