Linear Relations

Do you prefer a holiday that has an extra element of excitement? You may enjoy adventure travel tours, such as mountain biking, horseback riding, kayaking, or polar bear watching.

Whenever you travel, it is important to plan ahead. You need to think about such factors as the length of the trip, the cost, and the supplies you will need. One factor may be related to another—for example, a longer trip may cost you more. Sometimes you can show the relationship between these two factors mathematically.

What You Will Learn

- to recognize patterns and analyse data in a table of values
- to graph two-variable linear relations
- to solve problems using linear relations



Key Words

 relationship • table of values

expression

- variable formula
- linear relation
- equation

Literacy 🗧 Link

Before starting the chapter, copy the following KWL chart into your math journal or notebook. Brainstorm with a partner what you already know about linear relations.

- Record your ideas in the first column.
- List any questions you have about linear relations in the second column.
- As you complete each section of the chapter, list what you have learned in the third column.

For more information about how to use a KWL chart, go to Chapter 1 Literacy Link on page 3.

Linear Relations

What I Know	What I <i>Want</i> to Know	What I Learned

FOLDABLES™ Study Tool

Making the Foldable

Materials

- 11×17 sheet of paper
- seven sheets of centimetre grid paper
 or 0.5 cm grid paper
- stapler
- scissors

Step 1

Fold an 11×17 sheet of paper in half. Instead of creasing it, just pinch it at the midpoint. Fold the outer edges of the paper to meet at the midpoint.



Step 2

Staple two sheets of grid paper to the inside back of the Foldable. Label them 9.3 Linear Relationships, as shown.



Step 3

Fold five sheets of grid paper lengthwise in half. Cut one of the folded sheets in half.

Step 4

Insert two and a half of the sheets into the left crease of the Foldable. Insert two of the sheets into the right crease of the Foldable. Staple them into place.



Step 5

Label the outside of your Foldable with the labels Chapter 9 Linear Relations and What I Need to Work On, as shown in the diagram in Step 4.

Step 6

Put the following labels on the inside of your Foldable.

Fold on the Left

- First half page: Key Words.
- Front and back of second half page: Math Link introduction
- Third half page: 9.1 Analysing Graphs of Linear Relations
- Last two half pages: 9.1 Math Link

Fold on the Right

- First three half pages: 9.2 Patterns in a Table of Values
- Last half page: 9.2 Math Link

Using the Foldable

As you work through Chapter 9, make notes about the Key Words on the first half page inside the left flap.

Record your answers to the Math Link introduction on page 331 on the half page after the Key Words. Draw your graph on the back of that half page.

Record the examples and Key Ideas in the appropriate section. Use the grid paper to show examples of linear relations.

Place your answers to the Math Links in sections 9.1 and 9.2 in the appropriate sections. List your ideas for the Wrap It Up! on the back of the Foldable.

On the front of the right flap, keep track of what you need to work on. Check off each item as you deal with it.

MATH LINK Adventure Travel

Have you ever wanted to climb a mountain? Via Ferrata is a mountain experience for adventurers who want to climb to the summit of Whistler Peak in British Columbia. It features a vertical pathway with cables and metal ladders that lead climbers to the top.

Paulette decides to climb to Whistler Peak on the Via Ferrata trail. She times the last eight 100-m sections of her climb. Here are her times: 10 min, 9 min, 10 min, 11 min, 10 min, 30 min, 10 min, and 10 min.

- 1. Describe patterns you see in Paulette's data.
- **2.** Give two reasons why you think her sixth time was so different from all the others.
- **3.** Copy and complete the following table of values.

Total Distance (m)	Total Time (min)
100	10
200	19
300	
400	
500	
600	
700	
800	



- **4.** Draw a graph showing Paulette's total distance compared with the total time over the last 800 m of the climb. Label the axes.
- 5. Describe patterns you see on your graph.

Throughout this chapter, you will explore a variety of adventures and eco-travel opportunities. At the end of the chapter, you will plan an adventure or eco-travel package, showing mathematical relationships between quantities such as time, cost, and supplies.

What adventure do you want to go on?

Analysing Graphs of Linear Relations

Focus on...

After this lesson, you will be able to...

 describe patterns on the graph of a linear relation

create a table of values using the points on a graph

Literacy 🖯 Link

A *relationship* is a pattern formed by two sets of numbers.

Materials

- cardboard circle
- coloured counters



Graphs have been around for a long time. The pictorial writings of Aboriginal peoples and the drawings in the tombs of ancient Egypt are two examples. Graphs have gone through significant changes. Many graphs are now used to show relationships between sets of data.

A table of values also shows a relationship between two quantities. What is an advantage of a graph compared with a table of values?

Explore the Math

How can you represent a linear relation?

Tony's Pizzeria sells medium pizzas for \$9 each. You can order extra toppings for \$1 per topping.



The graph shows the cost of Tony's medium pizzas.



- **1.** a) Use a cardboard circle and coloured counters to model a medium pizza with extra cheese, pineapple, and ham.
 - **b**) Explain how your model could represent the cost of a medium pizza with three toppings. What cost does your model show?
- 2. Look at the graph. What is the cost of a medium pizza with three toppings? How does this compare with your answer in #1b)?
- **3.** a) From the graph, make a table of values that shows the cost of a medium pizza with zero to six toppings. ° ° ° 00
 - **b)** What headings did you use for your table? Why?

Reflect on Your Findings

- **4.** a) What are three ways you can represent data?
 - **b)** Which way do you prefer? Why?

Strategies Model It

Literacy Link

A table of values is a chart showing two sets of related numbers.

You can arrange a table of values horizontally or vertically. In a horizontal table, the top row should show the *x*-coordinates from a graph.



In a vertical table, the first column should show the *x*-coordinates from a graph.

У

5

10

15

20

X

1

2

3

4

Example 1: Make a Table of Values From a Graph

The graph shows the total cost in relation to the number of baseballs you buy.

- a) Describe patterns you see on the graph.
- **b)** Make a table of values from the graph.
- c) If the relationship continues, what is the cost of 14 baseballs?

a) The patterns can be described in the

• The graph provides data on the cost

of baseballs. One ball costs \$3, two

balls cost \$6, three balls cost \$9, ...The points appear to lie in a straight line.The graph shows that to move from one

Solution

following ways:





po	nt to the next, you	go one unit
ho	rizontally and three	units vertically.

b)	Number of Baseballs, b	1	2	3	4
	Total Cost, C (\$)	3	6	9	12

Strategies

Solve an Equation

 c) The graph shows that the cost increases by \$3 for each baseball purchased. Let *b* represent the number of baseballs. The cost could be represented by 3*b*.

Cost of 14 baseballs = 3(14)

$$= 42$$

The cost of 14 baseballs is \$42.

Show You Know

The graph shows the number of triangles in relation to the figure number in a pattern.

- a) Describe patterns you see on the graph.
- **b**) Draw a triangle pattern that matches the graph.
- c) Make a table of values from the graph.
- **d)** If the pattern continues, how many triangles are in Figure 99?





You can use a spreadsheet program to create a table.

Example 2: Analyse Data on a Graph of a Linear Relation

The graph shows Nicole's rate of pay based on the number of hours she has worked at her part-time job.

- a) Describe patterns you see on the graph.
- **b)** What is Nicole's hourly rate of pay? How do you know?
- c) Make a table of values from the graph.
- **d)** Is it possible to have points between the ones on the graph? Explain.

Solution

- a) The patterns can be described in the following ways:
 - The graph provides data on the pay Nicole receives for each hour worked. The pay for 1 h is \$10, the pay for 2 h is \$20, the pay for 3 h is \$30, ...
 - The points appear to lie in a straight line. The graph shows a linear relation.
 - The graph shows that to move from one point to the next, you go one unit horizontally and ten units vertically.
- b) Nicole's hourly rate of pay is \$10. The graph shows that Nicole's pay increases by \$10 for each hour that she works.

C)	Time, <i>t</i> (h)	0	1	2	3	4
	Pay, <i>P</i> (\$)	0	10	20	30	40

d) It may be possible to have points between the ones on the graph. For example, Nicole could get paid for working for $3\frac{1}{2}$ h, and then a point could be shown between (3, 30) and (4, 40) on the graph.









A *linear relation* is a pattern made by a set of points that lie in a straight line.



Key Ideas

- You can use the coordinates of the points on a graph to make a table of values.
- The top row or left column in a table of values has the same label as the horizontal axis. The second row or right column has the same label as the vertical axis.

Time Worked, t (h)	1	2	3
Pay, P (\$)	12	24	36

Time Worked, t (h)	Pay, <i>P</i> (\$)
1	12
2	24
3	36

- When you describe a pattern, tell what it relates to, where it starts, and how it changes.
- A linear relation is a pattern made by a set of points that lie in a straight line.







• Sometimes, it is possible to have points between the ones on a graph. Ask yourself, "Does it make sense to have other values between those on the graph?"

> Could there be points between the ones on the graph? For example, is it possible to determine the temperature at $2\frac{1}{2}$ min? Is it possible to read a temperature as part of a degree, such as 3.5 °C?

Communicate the Ideas

1. Tell whether you think it is reasonable to have points between the ones on each graph. Explain your answer.



- **2.** Draw a graph of a linear relation. Use integer values only. Label your graph. Write a brief description that matches the information on your graph.
- **3.** Use an example to show one way that a graph and a table of values are different and one way that they are similar.

Check Your Understanding Practise

For help with #4 and #5, refer to Example 1 on page 334.

4. The graph shows the increase in total height for each step of a staircase.



- a) Describe patterns you see on the graph.
- **b)** Make a table of values from the graph.
- c) If the pattern continues, what is the total height on step 10?

5. The graph shows the maximum number of students allowed on a field trip based on the number of teachers available to supervise.



- a) Describe three patterns on the graph.
- **b)** Make a table of values from the graph.
- c) If there are eight teachers available for a field trip, what is the maximum number of students who can go?

For help with #6 and #7, refer to Example 2 on page 335.

6. Tessa and Vince go shopping at Bulk Bin. The graph shows the cost of banana chips.



- a) Describe patterns shown on this graph. Does the graph show a linear relation? Explain.
- **b**) Make a table of values from the graph.
- c) Is it reasonable to include a point on the graph that shows the cost of 250 g of banana chips? Explain.
- **7.** The graph shows the height of a stack of cubes in relation to the number of cubes.



- a) Describe patterns on the graph. Does the graph show a linear relation? Explain.
- **b)** Make a table of values from the graph.
- c) Is it reasonable to include a point for c = 2.5? Explain.



8. a) Make a table of values for the ordered pairs on the graph.



- b) Assume the pattern continues. Extend your table, using the next three whole number values for x.
- c) Describe the patterns on the graph.
- d) What is the value of y when the value of x is 9?
- **9.** The graph shows the rate of pay based on the number of hours worked.



- a) Make a table of values from the graph.
- **b)** What is the hourly rate of pay shown on this graph?
- c) Do you think it is reasonable to include a point for t = 3.5 h?

10. The graph shows part of a linear relation that represents the cost to purchase sugar flowers for a cake.







- a) Is it reasonable to have points between the ones on the graph? Explain your answer.
- b) How many points could there be between the two shown on the graph? Explain your answer.
- The graph shows the simple interest for one year at 5% for different dollar amounts invested.



- a) What are the coordinates for point W?
- **b)** What does each coordinate for point W represent?
- c) Describe patterns on the graph.
- d) If the pattern continues, what is the simple interest earned on \$180 after one year?

12. The graph shows the perimeter of a square in relation to the length of its side.



a) Copy the table and fill in the missing values for *s* and *P*.

Side Length, s (cm)	Perimeter, P(cm)
0	0
1	4
2	
	12
4	
5	
	32
28	
	124

- **b)** Describe the patterns on the graph.
- c) Are any other points possible between those shown on the graph? Explain.
- **d)** Does the graph represent a linear relation? Explain.



13. The graph shows the cost of dried apricots at Bulk N Save.



- a) Make a table of values from the graph.
- **b)** Describe patterns on the graph.
- c) Use the graph to estimate the cost of 350 g of dried apricots.



- **d)** What is the actual cost of 350 g of dried apricots? Round your answer to the nearest cent.
- e) Compare your answers in parts c) and d).
- **14.** The graph below represents the relationship between the number of boxes of almonds a student sells for a school fundraiser and the profit in dollars for the school.



- a) Make a table of values from the graph.
- **b**) Describe three patterns on the graph.
- c) How much profit does the school make on two boxes of almonds?
- d) What is the value of P when the value of b is 2? How does this answer relate to your answer in part c)?

15. The following graph represents the number of words Tom can type in relation to the time in minutes.



- a) For the ordered pair (2, 80) tell what each coordinate represents.
- **b)** What is the typing speed in words per minute for point A?
- c) Does this graph represent a linear relation? Explain.
- **d)** Would most people's typing speed result in a linear graph? Explain.
- **16.** Alena gathered data comparing the amount of time she spent studying for her tests and the marks she received.



- a) Make a table of values from this graph.
- **b)** Does the graph appear to be a linear relation? Why?
- c) Will Alena's scores continue increasing at this same rate with more and more time spent studying? Why?



17. At Ciao Restaurant, Mario works in the kitchen for \$10 an hour. Susie works as a server. She gets \$30 monthly clothing allowance plus \$8 an hour.



- a) Which colour of points provides information about Susie's wages?
- b) Make a table of values with three columns showing Time Worked (h) and Total Pay (\$) for Susie and Mario.
- c) If the pattern continues, at what point will the two sets of points meet?

18. Mark begins saving \$15 a day to buy a three-month gym membership for \$90. Kendal has \$105 in the bank and spends \$5 a day to go to the gym. The graph shows how much money they have over the first three days.



- a) Which colour of points provides information about Mark?
- **b)** When will Kendal run out of money?
- c) When will Mark have saved enough money to buy the membership?

MATH LINK

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Whatever adventure you are looking for, it can be found in Western and Northern Canada. Adventures include polar bear tours in Churchill, Manitoba, aurora borealis adventures in the Northwest Territories, and white-water rafting in Yukon Territory.

You are going on a polar bear adventure tour. The graph shows the cost of the trip.

- a) Describe any patterns on the graph.
- **b)** Make a table of values from the graph.
- c) Assume the pattern continues. Extend your table of values to include the cost of a tour for eight days.
- d) Suppose it is possible to upgrade to better accommodations for a one-time fee of \$300 plus the regular \$400 per day. Make a new table of values for deluxe tours that last from one to eight days.
- e) Compare the data in the two tables of values. How are they similar? How are they different?



Patterns in a Table of Values

Focus on...

After this lesson, you will be able to...

- identify relationships in a table of values
- decide if a table of values represents a linear relation
- graph points represented by values in a table



As humans, we see patterns all around us. It is a way for us to make sense of our world. However, different people may see patterns in different places and in different ways. In the picture, Kendra describes a pattern she sees in the wallpaper. What patterns can you identify?

Explore the Math

How can you represent patterns?



- 1. What are three patterns you see in the design?
- 2. How many squares and triangles are in this design?
- **3.** The design can be lengthened or shortened. Each section of the design must have only complete squares and triangles. If this design has ten sections, what does one section of the design look like?



4. Make a table of values like the one shown. Complete your table for one to four sections in the design.

Number of Sections	Number of Vertical Segments	Number of Horizontal Segments	Number of Slanting Segments
1			
2			
3			
4			

- **5.** a) Use words to describe the relationship between the number of sections and the number of vertical segments.
 - **b**) Choose variables to represent the number of sections and the number of vertical segments. Why did you choose these letters?
 - c) Draw a graph to show the relationship between the number of sections and the number of vertical segments. Label the axes with the variables you chose in part b), and with the titles Number of Sections and Number of Vertical Segments.
 - d) Describe in words the relationship between the variables.
 - e) What is an expression for the number of vertical segments in terms of the number of sections?
- 6. a) Draw a graph to show the relationship between the number of sections in the design and the data in one of the other columns in the table. Label each axis with a variable and a title.
 - **b)** What is an expression that represents the data in the column you chose in terms of the number of sections in the design?

Reflect on Your Findings

- **7.** a) What ways did you use to represent patterns in this activity?
 - **b)** Which parts of the activity did you find most challenging? Why?

WWW Web Link

To practise your graphing skills, go to www.mathlinks8.ca and follow the links.

A variable is a letter that represents an unknown quantity. For example, in 3a - 5, the variable is a.

Literacy 🔁 Link

It can be helpful to choose variables that are meaningful. For example, t for time and s for score.

Literacy 🔁 Link

An expression is any single number or variable, or a combination of operations $(+, -, \times,$ ÷) involving numbers and variables. An expression does not include an equal sign. The following are examples of expressions: 5 r 8t

x + 9 2y - 7

0000 vertical horizontal slanting / or \

Example 1: Identify the Relationship in a Table of Values

The pattern in the table of values represents a linear relation.

- **a)** Graph the ordered pairs in the table of values.
- b) What is the difference in value for consecutive A-values? for consecutive B-values?

Α	В
0	0
1	3
2	6
3	9
4	12

- c) Describe in words the relationship between the values for A and B.
- d) What is an expression for *B* in terms of *A*?

Solution



Mentally check with values for A in the table. $3 \times 1 = 3$ $3 \times 2 = 6$ $3 \times 3 = 9$ $3 \times 4 = 12$



You can use a spreadsheet program to make the graph.

- **c)** When *A* increases by 1, *B* increases by 3.
- d) Use the difference in *B*-values of 3. Each *B*-coordinate is 3 times its *A*-coordinate.

An expression for *B* is $3 \times A$, or 3A.

Show You Know

The table of values represents a linear relation.

x	1	2	3	4	5	6	7
У	5	10	15	20	25	30	35

- a) Graph the ordered pairs in the table of values.
- **b)** What is the difference in value for consecutive *x*-values? What is the difference in value for consecutive *y*-values?
- c) Describe in words the relationship between the *x*-values and *y*-values.
- d) What is an expression for y in terms of x?

b) The difference in consecutive *A*-values is 1. Find the difference in consecutive *B*-values.

3 - 0 = 36 - 3 = 39 - 6 = 312 - 9 = 3

The difference in consecutive *B*-values is 3.



Example 2: Use a Table to Determine a Linear Relation

For each table of values below, answer the following questions:

- a) What is the pattern in the values for the first variable in each table?
- **b)** What is the difference in consecutive values for the second variable in each table? Is the difference within each table the same?
- c) Graph each set of ordered pairs. Which relations are linear?
- d) How does your answer in part c) compare with your answer in part b)?

Solution

- a) Table 1: The *x*-coordinates differ by 2. Table 2: The *m*-coordinates differ by 1. The consecutive values of the first variables differ by the same amount within each table.
- **b)** Table 1: The *y*-coordinates differ by 4. The difference is the same. Table 2: The *n*-coordinates differ by 3, 3, and 1. The difference is not the same.



The relation is linear.



 Table 1

 x
 2
 4
 6
 8

 y
 3
 7
 11
 15

Table 2						
m	1	2	3	4		
n	1	4	7	8		



The relation is not linear.



If consecutive values for the first variable do not have the same difference, it is difficult to tell from the table whether the relationship is linear. You may be able to tell by drawing a graph.

d) Table 1: The difference in the *y*-coordinates is the same and the relation is linear.

Table 2: The difference in the *n*-coordinates is not the same and the relation is not linear.

Show You Know

Determine whether each table represents a linear relation. Explain how you know. Check each answer by graphing the ordered pairs.

Table 1

Α	1	4	7
В	1	10	19

Tab	le 2
-----	------

р	0	1	2	3
q	0	1	3	6

Example 3: Use a Table of Values in Solving a Problem

Photo World charges \$3 for the first enlargement and \$2 for each additional enlargement.

- a) Make a table of values showing the cost in relation to the number of enlargements for one to five enlargements.
- **b)** Is this a linear relation? Why?
- c) What is an expression for the cost in relation to the number of enlargements?
- d) What is the cost of 15 enlargements?

Solution

a)	Number of Enlargements, n	1	2	3	4	5
	Cost, C (\$)	3	5	7	9	11

b) The difference between consecutive numbers of enlargements is 1. Determine the difference for consecutive costs.

$$5 - 3 = 2$$

$$/ - 3 = 2$$

$$9 - 7 = 2$$

 $11 - 9 = 2$

Consecutive costs differ by 2.

Each consecutive number of enlargements differs by the same value. Each consecutive cost differs by the same value.

The relation is linear.

c) The cost increases by \$2 for each additional enlargement.

Let *n* represent the number of enlargements.

2n will be part of an expression for the cost.

Check using a value for *n* from the table: n = 4.

2(4) = 8

The value of 8 is not the same as the value of 9 in the table. You need to add 1 to get from 8 to 9.

An expression for the cost is 2n + 1, where *n* is the number of enlargements.

d) Substitute n = 15 into the expression 2n + 1. 2(15) + 1 = 31

The cost of 15 enlargements is \$31.

Mentally check M = E 2n + 1 with other values for *n* in the table. 2(1) + 1 = 3 2(2) + 1 = 5 2(3) + 1 = 72(5) + 1 = 11

Show You Know

Sky sells magazine subscriptions. She receives \$20 for every five subscriptions she sells. The table shows the relationship between the number of subscriptions she sells and the pay she receives.

Number of Subscriptions, n	0	5	10	15	20	25
Pay, P (\$)	0	20	40	60		

- a) Copy and complete the table.
- **b)** Is this a linear relation? How can you tell?
- c) What is an expression for Sky's pay in relation to the number of subscriptions she sells?
- d) How much does Sky receive for selling 40 subscriptions?

Key Ideas

- The following are some ways that you can represent a linear relation.
 - Table of values:

Mass, m (g)	0	100	200	300
Cost,C(¢)	0	200	400	600

- Words: The cost in cents is 2 times the mass in grams.
- Ordered pair: (m, 2m)
- Expression: The cost in cents is 2*m*, where *m* is the mass in grams.
- 200 0 Mass (g)

19

• Graph:

• You can sometimes tell from a table whether the relationship is linear.

80

	Table 1						le 2
p	2	3	4	5		р	q
q	7	13	19	25		20	3
						40	2
						60	2

You can tell that the relationships in the above tables are linear because both of the following statements are true:

- Each consecutive value for *p* changes by the same amount.
- Each consecutive value for *q* changes by the same amount.



Communicate the Ideas

1. Giselle and Tim are discussing the table of values shown.

m	3	5	7	9
а	1	3	5	7

Who is correct? How do you know?

 a) Describe a real-life situation that matches the pattern in the table of values.

n	1	2	3	4
р	1	3	5	7

- **b**) What do *n* and *p* represent in your situation?
- **3.** You are given a table of values. Can you sometimes, always, or never tell whether the relationship is linear without drawing the graph? Use examples to support your answer.

Check Your Understanding

Practise

For help with #4 to #7, refer to Example 1 on page 344.

4. Graph the ordered pairs in the table of values.

а	d
1	5
2	8
3	11
4	14

5. Draw a graph using the ordered pairs in the table of values.

w	t
1	1
4	7
7	13
10	19

6. The table of values represents a linear relation.

The table does not represent a linear relation.

x	0	1	2	3	4	5
а	0	4	8	12	16	20

I think

it does

- a) Graph the ordered pairs.
- b) What is the difference in value for consecutive *x*-values? What is the difference in value for consecutive *a*-values?
- c) In words, describe the relationship between *x* and *a*.
- d) What is an expression for *a* in terms of *x*?

7. The table of values shows a linear relation.

n	3	4	5	6	7
d	18	24	30	36	42

- a) Graph the ordered pairs.
- b) What is the difference in value for consecutive *n*-values? What is the difference in value for consecutive *d*-values?
- c) In words, describe the relationship between *n* and *d*.
- d) What is an expression for *d* in terms of *n*?

For help with #8 and #9, refer to Example 2 on page 345.

8. For each table of values, tell whether the relationship is linear. Explain your answer. Check by graphing the ordered pairs.

a)	c	d
	2	7
	3	10
	4	13
	5	16

b)	x	у
	0	-3
	1	1
	3	9
	5	17

9. Tell whether the relationship for each table of values is linear. Explain how you know.

a)	р	4	7	10	16
	q	11	17	23	29
b)	x	2	3	4	5
	у	3	2	1	0

For help with #10 and #11, refer to Example 3 on page 346.

- **10.** Mara reads at a rate of 90 words per minute.
 - a) Make a table of values that shows the total number of words Mara reads in one to six minutes. Use whole minutes.
 - **b)** Is this a linear relation? Explain.

- c) What is an expression for the number of words Mara reads in terms of time?
- d) How many words does Mara read in 12 min?
- 11. The dosage of a certain medication to be given to a child is related to the child's mass. A 10-kg child receives 50 mg of the medication. For each increase in mass of 1 kg, the child receives an additional 10 mg of the medication.
 - a) Make a table showing the dosage for children with an increase of 1 kg to 10 kg over the mass of 10 kg. Use whole kilograms only.
 - **b**) Is this a linear relation? Explain how you know.
 - c) What is an expression for the dosage in terms of the increase in mass over 10 kg?
 - d) What is the dosage for a child with a mass of 27 kg?
 - e) Can your table of values start at 0 kg? Why or why not?

Apply

12. Evan has \$6 in quarters and dimes.



- a) Name five combinations of quarters and dimes that Evan might have.
- **b)** Make a table of values showing the relationship between quarters and dimes. Include five pairs of values in your table.
- c) Draw a graph. Is the relationship between quarters and dimes linear in this example? Explain.
- **d)** What is the largest possible number of dimes? of quarters?

13. Divers experience an increase in pressure as they dive deeper below sea level. The table shows the relationship between depth in metres and pressure in atmospheres.

Depth (m)	Pressure (atm)
0	1
10	2
20	3
	4
	5
	6

- a) Copy and complete the table.
- **b)** Draw a graph for the ordered pairs in the table. How should your axes be labelled?
- c) Divers sometimes become dizzy when the pressure exceeds 5 atm. Below what depth do divers tend to become dizzy?
- **14.** The following pattern continues.



- a) Make a table of values showing the figure number and the number of squares for the first six figures.
- **b)** Write an expression showing the number of squares in terms of the figure number. What does your variable represent?
- **c)** How many squares would appear in Figure 20?
- d) How many more squares are in Figure 20 than in Figure 10? Show two ways to find the answer.

15. The following pattern of squares continues.



a) Copy and complete the table of values below that shows the relationship between the number of squares and the perimeter of each figure.

Number of Squares	1	2	
Perimeter (cm)	4	6	

- **b)** Draw a graph from the table of values.
- c) Describe the patterns on the graph.
- **d)** What is an expression for the perimeter in terms of the number of squares?
- e) If the pattern continues, what is the perimeter when there are 50 squares?
- **16.** As you climb a mountain, the temperature drops 1 °C for every 150 m of increased height.
 - a) Copy and complete the table to show the relationship between height and temperature if the temperature at the bottom of the mountain is 20 °C.

Height (m)	0	150	300	450	600	750
Temperature (°C)	20					

- **b)** Graph the ordered pairs
- c) Is the relationship linear?
- **d)** How high have you climbed if the temperature is 13 °C?
- **17.** A skydiver jumps from an airplane. The table provides data for the period of time shortly after the parachute opens, relating time in seconds to total distance descended in metres.

Time (s)	Distance (m)
10	300
11	354
12	408
13	462



- a) Predict how a graph for these ordered pairs would look.
- **b)** Graph the ordered pairs. Was your prediction correct?
- c) Describe the graph in words.
- **18.** A community centre has a new banquet hall. The centre charges \$5 per person to rent the hall.
 - a) Make a table of values showing the rental cost for 20, 40, 60, 80, and 100 people.
 - **b)** Graph the ordered pairs.
 - c) What is an expression for the rental cost in terms of the number of people?

Extend

- **19.** The community centre in #18 changes the cost for renting its banquet hall. The centre now charges \$50 plus \$5 per person.
 - a) Make a table of values showing the rental cost for 20, 40, 60, 80, and 100 people.
 - **b)** Graph the ordered pairs. How does the graph differ from the graph in #18?

- c) What is an expression for the rental cost in terms of the number or letter of people? What does each number or letter in your expression represent?
- **20.** Jamal is renting snowboard equipment.



a) Copy and complete the table.

Number of Additional Days	0	1	2	3	4	5
Rental Cost (\$)						

- **b)** What is an expression for the rental cost in terms of the number of additional days?
- c) What is the cost to rent the snowboard for a total of ten days? What might be a better option for Jamal instead of renting for ten days?

MATH LINK

Have you ever gone for a canoe trip on the waters of one of Canada's national parks? You are planning a canoe trip. The cost to rent a canoe is \$40 a day. A national park pass for one week costs \$36.

- a) Make a table of values showing the total cost for the pass and canoe for a trip from one to seven days.
- **b)** Graph the ordered pairs in your table of values. Is this a linear relation? Explain.
- c) What is an expression for the cost based on the number of days?
- d) Think of a linear relationship to do with canoeing. Show the relationship using a table of values, a graph, words, and an expression.



Linear Relationships

Focus on...

J

After this lesson, you will be able to...

- graph from a formula
- graph from an equation using integers
- solve problems using the graph of a linear relation



Scientists look for patterns. They collect experimental data in charts and tables. Then, they graph the data. These graphs are pictures that show mathematical relationships in the data. When scientists write a formula for the relationship, it may represent a new scientific discovery! What are some formulas that you are familiar with?

Explore the Math

How do you determine a relationship?

Work in a group.

1. Beginning in a corner, place about 2 m of masking tape horizontally on a wall at eye level. Start at the corner and mark off the masking tape in 5-cm intervals.



Materials

- metre stick or measuring tape
- masking tape
- different-sized tubes such as from paper towel rolls
- grid paper 💿

- **2.** On the floor, measure and mark five distances from the wall. Start at 100 cm and increase the distance by 30 cm each time.
- **3.** Copy the table into your notebook.

Distance From Wall (cm)	Width of Field of View (cm)

- **4.** Predict how the width of your field of view through a tube will change as your distance from the wall increases.
- **5.** For each distance from the wall, measure the width that you can see through the tube. Record the data in the table.
- **6.** Write the results of your experiment as a set of ordered pairs. Graph the ordered pairs.
- 7. Compare your results with your prediction.
- 8. Repeat steps 3 to 7, using a different-sized tube.

Reflect on Your Findings

- 9. a) Do your data show a linear relation? Explain.
 - **b)** Why might the points on your graph line up as they do?

Example 1: Graph From a Linear Formula

Sound travels 1500 m/s in salt water. This relationship can be expressed by the formula d = 1500t, where d is the distance travelled, in metres, and t is the time, in seconds.

- a) Make a table of values. Use integers only.
- **b**) Graph the ordered pairs in your table of values.
- c) Is it reasonable to have points between the ones on this graph? Explain.
- d) Calculate the distance sound travels in 3.5 s.

Literacy <mark>S Link</mark>

A formula is a mathematical statement that represents the relationship between specific quantities. An example is $C = \pi \times d$, where C is the circumference and d is the diameter of the circle.

Solution

a) Choose the numbers to use in the table of values. For example,



- c) It is reasonable to have points between the ones on the graph. For example, it is possible to determine the distance at 2.5 s, and it is possible to determine the time at 4800 m.
- d) Substitute 3.5 for t in the formula d = 1500t.
 - d = 1500(3.5)
 - d = 5250

The distance travelled at 3.5 s is 5250 m.

The estimate of 5000 m is close to the calculated distance of 5250 m.

Show You Know

Paula rents a lawnmower for \$8 per hour. A formula representing this relationship is C = 8t, where C is the rental cost, in dollars, and t is the time, in hours.

- a) Make a table of values.
- **b**) Graph the ordered pairs in your table of values.
- c) Is it reasonable to have points between those on the graph? Explain.
- d) Calculate the cost to rent for 12 h.



Example 2: Graph From a Linear Equation Using Integers

Use the linear equation y = -3x + 4 to complete the following steps.

- a) Make a table of values. Use positive and negative integers and zero for values of *x*.
- **b**) Graph the ordered pairs in your table of values.
- c) Determine the value for y in the ordered pair (11, y).

Solution

a) Choose the numbers to use in the table of values. For example, o



c) Substitute x = 11 into the equation y = -3x + 4. y = -3x + 4y = -3(11) + 4

- y = -33 + 4
- y = -29

The value of the *y*-coordinate is -29. The ordered pair is (11, -29).

Show You Know

Use y = 2x + 3 to answer the following.

- a) Make a table of values. Use integers only.
- **b)** Graph the ordered pairs in your table of values.
- c) What are the coordinates for the point that would lie on the y-axis?

What is the value for x of a point that lies on the y-axis?



An equation is a mathematical statement with two expressions that have the same value. The two expressions are separated by an equal sign. For example, x + 2 = 3y - 7 = -4

3a - 2 = a + 2

b = 4

Since no context is given, many

values for x and y are possible.

Choose values for x that are the same distance apart. For example, you could choose x = -1, 0, 1, 2, 3, 4 or

x = -2, 0, 2, 4, 6, 8.

Solve an Equation

Strategies

0

Key Ideas

- You can graph a linear relation represented by a formula or an equation.
 - First, make a table of values. Check that the values in the table are reasonable.
 - Then, graph using the ordered pairs in the table.

$$t = 3h - 2$$

• Whenever possible, choose variables that are meaningful. For example, *h* for height and *t* for temperature.



Communicate the Ideas

- 1. a) What do you notice about the coordinates for points on the *x*-axis?b) What do you notice about the coordinates for points on the *y*-axis?
- **2.** The equation y = 2x 1 represents a linear relation. A table of values for y = 2x 1 has been started below. Show two different ways to find the value of the missing *y*-coordinate.

x	0	1	2	3	4
у	-1	1	3	5	

- **3.** a) When you choose values for a table of values, what considerations might influence your choice?
 - **b)** Why is it often useful to use zero as one of your *x*-values?
- 4. a) Give an example of a linear relation found in real life. Then, give an example of a linear relation without a context.
 - **b**) Make a table of values for each linear relation. Use integers only.
 - c) Graph each set of ordered pairs.
 - d) How are these graphs similar? How are they different?

Check Your Understanding

Practise

For help with #5 and #6, refer to Example 1 on pages 353–354.

5. A long-distance telephone plan can be represented as C = 6t, where C is the cost in cents and t is the time in minutes.



- a) Make a table of values, using at least four whole number values for *t*.
- **b)** Graph the ordered pairs.
- c) If part minutes are rounded up to the next whole minute, is it reasonable to have points between the ones on your graph? Explain.
- 6. An animal shelter hires you to walk dogs for \$5 per dog. The relationship between the money you make and the number of dogs you walk can be expressed as W = 5d, where W is the wage you make, in dollars, and d is the number of dogs you walk.



- a) Make a table of values.
- **b)** Graph the ordered pairs.
- c) Is it reasonable to have points between the ones on your graph? Explain.

For help with #7 to #12, refer to Example 2 on page 355.

- **7.** Evaluate each equation using the given value.
 - **a)** y = 5x 3 when x = 6
 - **b)** y = x 8 when x = 5
 - c) y = -5x when x = -2
 - **d**) y = x when y = 25
- **8.** Evaluate y = 7x + 3 using each value.
 - a) x = 1b) x = -4c) x = 0d) y = 17
- 9. Make a table of values for each equation using x = -2, 0, 2, 4. Draw each graph.
 a) y = 3x + 2
 b) y = x 5
 c) y = -4x
 d) y = 7 x
- 10. Make a table of values for each equation. Choose at least four integer values for *x*. Graph each set of ordered pairs.
 - a) y = 2xb) y = 3x - 1c) y = -4x + 5d) y = 5 - x
- **11.** The graph below represents part





- a) Use the equation to calculate the *y*-coordinate when x = -1.
- **b)** What is the value of y in (-4, y)?

12. The graph represents part of the linear relation $y = \frac{x}{3}$.



- a) What are the coordinates for the point that lies on the *y*-axis?
- **b)** Use the equation to calculate the *y*-coordinate when x = -3.
- c) For the point (-9, y), what is the value of y?

Apply

13. a) Graph the ordered pairs in the table.

x	У
-2	0
1	2
4	4
7	6

- **b)** Is it reasonable to assume there are points between the ones on your graph if you have no other information? Why?
- **14.** You are given part of the table of values for a linear relation.

x	-3	-2	-1	0	1	2
у				6	8	10

- a) How could you use a pattern to find the missing *y*-coordinates?
- **b)** Identify the missing *y*-coordinates.

- In a bulk food store, rice crackers sell for 80¢ per 100 g.
 - a) Copy and complete the table of values.

Mass of Purchase (g)	Cost (¢)
0	
100	
200	
300	

- b) If you continue the table, what is the next most logical value to use for Mass of Purchase? Explain, using your knowledge of linear relations and patterns.
- c) Graph the ordered pairs.



16. The graph shows Nigel's monthly earnings.



- a) If Nigel does not make any sales, what are his monthly earnings?
- b) Nigel has sales of \$4000 in one month. How much does he earn?
- c) Nigel earns \$1500 in a month. What are his sales?

- **17.** You can buy work gloves from The Fix-It Store's web site according to the formula C = 5g + 2, where *C* is the cost in dollars and *g* is the number of pairs of gloves.
 - a) Make a table to show the number of pairs of gloves purchased in relation to the total cost. Use five values for *g*.
 - **b)** Graph the ordered pairs.
 - c) Does the relation appear linear? Why?
 - d) Are there other points possible between the ones on the graph? Explain.
 - e) What might 2 represent in the formula?
- 18. George is a carpenter. He has a rewards card for the hardware store he uses. He receives 40 points for every \$1 he spends. He can use the points to get savings on future purchases.
 - a) Copy and complete the table showing the dollar amount spent in relation to the number of points received.

Amount Spent (\$)	Points Received
1	40
2	
3	
4	
5	

- **b)** How many points does George receive for spending \$100?
- c) George can get a hammer for 100000 points. How much money does he have to spend to get the hammer?



19. Shandi determined the mass of five pieces of a type of metal. The table shows her results. The relationship is linear. Shandi made one error in finding the masses.

Volume (cm ³)	8	9	10	11	12
Mass (g)	88	99	110	121	144

- a) Use patterns to show how to find the incorrect mass. What is the correct mass?
- **b)** Draw points on a graph using Shandi's results in the table above.
- c) How could you use your graph to show which value is incorrect?

Extend

- **20.** A taxi company in Edmonton charges \$3 for the first 210 m plus 20¢ for each additional 210 m.
 - a) What is the cost for a trip of length 2100 m? of length 4.41 km?
 - **b)** Make a table of values showing the relationship between the distance in metres and the cost in cents. Use integers only.
 - c) Graph the ordered pairs.
 - d) Is it a linear relation? Why?
- **21.** Describe the pattern in each table of values. Then, graph each set of ordered pairs.

a)	t	0	1	2	3	4
	d	2	3	4	5	6
b)	x	8	7	6	2	
	у	4	3	2	-2	

22. a) Complete the table using the relationship "multiply *x* by 2 and then add 3 to get *y*."

x	-2	-1	0	1	2	3	11
у							

b) Is it a linear relation? Explain.

Chapter Review

Key Words

Unscramble the letters for each term. Use the clues to help you.

- **1.** P R E S E X N I O S an example is n - 4
- **2.** EILRAN OTLERAIN

a pattern in which the points lie in a straight line (two words)

3. M A L U R O F

an equation that represents the relationship between specific quantities

4. Q O N A T U I E

a mathematical statement with two expressions that have the same value

- **5.** A B E V I R A L in 3A + 2, the letter A is an example
- 6. L E B T A F O S U A V L E

a table showing two sets of related numbers (three words)

9.1 Analysing Graphs of Linear Relations, pages 332–341

Р

45

36 •

9

0

<u>@</u> 27 ·

Age 18.

Klaus's Rate

of Pav

Time (h)

- **7.** Klaus works after school. The graph shows his rate of pay.
 - a) Make a table of values from the graph.
 - **b)** Does the graph represent a linear relation? Explain.
 - c) Is it possible to have other points between the ones on this graph? Explain.

- **8.** The graph shows a linear relation.
 - a) Describe what the graph is about.
 - **b)** Describe patterns on the graph.



Grade 8 Car Wash

- c) What is the cost of one car wash?
- d) Make a table of values from the graph.

Ι

50

40

e) If 15 cars are washed, what is the income for the grade 8 class?



- **9.** The graph shows part of a linear relation.
 - a) Describe patterns on the graph.
 - **b)** Make a table of values using at least five whole number values for *x*.



- c) What is the value of y when x = 2?
- d) What is the value of y when x = 5?

9.2 Patterns in a Table of Values, pages 342–351

- **10.** The table of values represents a linear relation.
 - a) Graph the ordered pairs.
 - b) What is the difference in value for consecutive A-values? What is the difference in value for consecutive B-values?

Α	B
0	1
1	5
2	9
3	13
4	17
5	21

- **c)** Describe the relationship between the values for *A* and *B*. Use words and an expression.
- **11.** For each table of values below, answer the following questions.
 - a) What is the difference in consecutive values for the first variable?
 - **b)** What is the difference in consecutive values for the second variable? Is the difference the same for consecutive values?

Table 1

c) Graph the ordered pairs to check your answer.

m	-2	-1	0	1	2	3
n	-4	-2	0	2	4	6
Ta	ble 2		Ta	ble 3		
р	q		d	С		
-4	9)	1	5		
-2	5		2	8		
0	1		3	10		
2	-3	;	4	13		
4	-7	7	5	15		

- **12.** Speedy Print Shop charges \$2 for the first colour copy and \$1 for each additional colour copy.
 - a) Make a table of values representing the number of colour copies in relation to the cost. Include zero to five colour copies.

- **b)** Is this a linear relation? Explain.
- c) What is an expression for the cost in terms of the number of colour copies?
- d) What is the cost of 12 colour copies?

9.3 Linear Relationships, pages 352–359

- **13.** Craig travels at a constant speed in kilometres per hour. The formula d = 15t represents the relationship.
 - a) What does each variable represent?
 - **b)** What does 15 represent?
 - **c)** Make a table of values. Use five whole number values for *t*.
 - d) Graph the ordered pairs.
 - e) Is it reasonable to have points between the ones on the graph? Explain.
 - f) How far would Craig travel in 8 h?



14. For each equation, make a table of values using five positive and negative integer values for *x*. Graph the ordered pairs. Then, determine the value for *y* when x = -7.

Equation A: y = 7xEquation B: y = 3x - 2Equation C: y = -2x + 3

- **15.** Dana has graphed the equations y = 2x + 1 and y = -2x + 1, using integer values.
 - a) How are the graphs similar?
 - **b)** How are they different?

Practice Test

For #1 to #5, select the best answer.

- **1.** You can describe 2x 1 as a(n)
 - A constant
- **B** equation
- **c** expression
- **D** variable
- **2.** The table shows the toothpicks in the base of a triangle in relation to its perimeter.

Toothpicks in Base (<i>b</i>)	Toothpicks in Perimeter
1	3
2	6
3	9



Which expression represents the number of toothpicks in the perimeter of any triangle in this pattern?

A b + 3**c** $\frac{b}{3}$

B
$$3b$$

D $b-3$

3. Which table of values represents the linear relation shown?



3

2



6

4

4. Which table of values represents the linear equation y = 3x - 2?



У
2
8
1
1
1
<u>у</u>
y 9
y 9 15

5. Which graph represents "a banquet room rents for \$50 plus \$2 per person"?



Complete the statements in #6 and #7.

- **6.** If the equation is s = -4t + 2, the value for s in (-1, s) is \blacksquare .
- 7. To describe the graph in #3, you can say that when the *x*-coordinate increases by 1, the *y*-coordinate by .

У

8

Short Answer

8. The graph shows the cost of a new drink called Zap.



- a) What is the price per can of Zap?
- **b)** Describe three patterns on the graph.
- **c)** If you placed a point at (0, 0), what would each coordinate represent?

9. The pattern can be represented by the formula b = 4f, where b is the number of black dots and f is the figure number.



- a) Make a table of values for the number of black dots in Figures 1 to 5.
- **b**) Use the formula to determine the number of black dots in Figure 60.

Extended Response

10. The formula for the pattern below is s = 2f + 1, where *s* is the number of small squares and *f* is the figure number.



- a) Make a table of values for the first five figures in the pattern.
- **b)** Draw a graph to show the relationship.
- c) Is the relationship linear? Explain.

WRAP IT UP!

You are going on an adventure tour. Your adventure could be hang-gliding, hiking, canoeing, white-water rafting, dog sledding, whale watching, cycling, or any other adventure that interests you. What is your adventure? Where does it take place?

Use travel brochures, the Internet, or other sources to locate information on your adventure. Then, find or create data for a linear relation that has to do with your adventure. Use integers only. Refer to the Math Links in this chapter for ideas.

- a) Write one or two paragraphs giving information on your adventure.
- **b)** Arrange the data for your linear relation in a table of values.
- c) Graph the ordered pairs listed in your table values.
- d) Is it reasonable to have points between the ones on your graph? Explain why or why not.





Friends and Relations

- **1.** The Friends and Relations game is played by two friends and involves relations. These are the rules:
 - Each player spins the spinner once to decide who will deal the cards. If there is a tie, spin again.
 - The dealer shuffles and deals ten Friends and Relations game cards to each player. The other player takes the first turn.
 - For each turn, flip over the top card in your stack and spin the spinner.
 - Copy the linear relation from the card into the Linear Relation column of your Friends and Relations record sheet. Record the result of the spin in the *x*-column.
 - Calculate the *y*-value by substituting the *x*-value into the linear relation. Record the *y*-value in the *y*-column of the record sheet.
 - After each turn, record your total score, which is the sum of all the *y*-values you have recorded so far.
 - Check each other's calculation of each *y*-value and each total score.
 - The player with the higher total score after ten turns is the winner.
 - If there is a tie after ten turns, shuffle the deck again and deal the cards. Take more turns until one player pulls ahead.
- **2.** Play variations on the game. Here are some possible variations to get you started:
 - Determine the integer *x*-values in different ways. For example, you might design a different spinner or roll dice.
 - Make your own cards that show linear relations. (Make sure that they will result in integer *y*-values for integer *x*-values.)

Materials

- spinner with 9 sectors (numbered with integers
 - from 4 to 4) 💿
- paper clip (to be used with the spinner)
- set of 20 Friends and Relations game cards
- Friends and Relations record sheet for each student

My y-value was 14 in my first turn and -8 in my second turn. My total score after two turns is 14 + (-8), which equals 6.



Linear Relation	x	y	Total

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Comparing Wages

Five people work at Moy's Food Mart. One of the weekly time sheets is displayed below.

Employee	Mon	Tues	Wed	Thurs	Fri	Sat	Sun	Total Hours	Hourly Wage	Total Wage
Mr. Moy	8	8	8	8	9	-	_		\$12.50	
Ms. Wong	_	8	8	7	-	9	9		\$9.50	
Maria	8	5	-	_	6	9	5		\$7.50	
Tom	4	4	4	4	4	8	8		\$7.50	
Jacob	-	-	5	7	7	5	5		\$7.50	
							Total			

- **1.** a) For each employee, calculate the total hours and total wage.
 - **b)** What total amount is paid out in wages for the week?
- **2.** Which day is the busiest day of the week? the slowest? Justify your choices.
- 3. a) What is the average hourly wage for store employees?
 - b) There is more than one way to answer part a).Show a second method.
 - c) Which method gives a better indication of the average hourly wage? Explain.
- **4.** Draw a graph showing how much Mr. Moy, Ms. Wong, and Maria would make for 0, *5*, 10, 15, and 20 h of work. Use a different colour for each person.
- **5.** Maria has been offered a 40% raise and a promotion to manager.
 - a) Complete a table of values for Maria's new wage using 0, 5, 10, 15, and 20 as the number of hours worked.
 - **b)** Plot Maria's new wage on the graph from #4 using a different colour.
 - c) List the following in order according to their hourly wage, from highest to lowest: Mr. Moy, Ms. Wong, Maria before her raise, and Maria after raise.
 - d) Describe, in words, how the points on the graph are plotted with respect to the employees' hourly wage.



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